TE and TM modes polaritons in multilayer system comprise of a PML-type magnetoeletric multiferroics and ferroelectrics

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TE and TM modes polaritons in multilayer system comprise of a PML-type magnetoelectric multiferroics and ferroelectrics

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Abstract. In this paper, we report our study on both bulk and surface polaritons generated in Multilayer system. The multilayer consists of ferroelectric and multiferroic with canted spins structure. The effective medium approximation is employed to derive the dispersion relation for both bulk and surface modes. Surface and bulk polaritons are calculated numerically for the case of Transverse electric (TE) and Transverse magnetic (TM) modes. Example results are presented using parameters appropriate for $BaMnF_4/BaAl_2O_4$. We found in both TE and TM modes, that the region where the surface modes may exist is affected by the volume fraction of the multiferroics. The region of the surface modes decrease when the volume fraction of the multiferroic is reduced. This region decrement suppress the surface polariton curves which result in shortening the surface modes curves.

1. Introduction

Electromagnetic waves which are coupled to the elementary excitations of the materials are called polaritons[1]. Polaritons have received much attention due to its unique properties. Surface polaritons are able to demonstrate localization on the surface. They can also show the non reciprocity, where the two opposite directions of propagation have different frequencies, i.e. $\omega(k) \neq \omega(-k)$[2–4].

Theoretical and experimental treatments for phonon polaritons in dielectrics were published several decades ago[5–7]. The theoretical studies of magnon polaritons in magnetic materials were also reported several years ago [3, 8, 9] and reviewed in Ref.[4]. Experimental studies of magnon polaritons in antiferromagnet had been performed using many methods: transmission for FeF_2[10],reflection for CoF_2[11], MnF_2[12] and attenuated total reflection (ATR)[13].

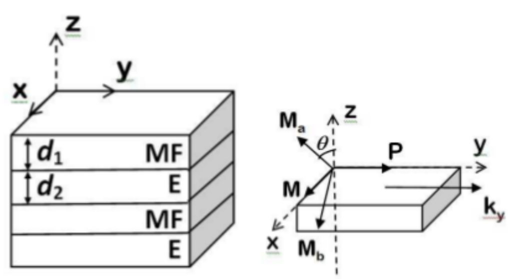
The study of polaritons in multiferroics is interesting since multiferroics have both phonon polaritons and magnon polaritons simultaneously[14–17]. It is happen since in multiferroics exist at least two types of ferroicity[18], hence there will be more than one resonance frequency in the multiferroics. For example, magnetoelectric multiferroics have both phonons and magnons which lead to the existence of the electric and the magnetic resonance frequencies simultaneously.

Study the multilayer geometry by combining at least two types of different material is important to understand the changes of the elementary excitations of the materials which are represented by the changes of the polaritons. The manipulation of polaritons had been studied theoretically by considering the multilayer configuration which comprise of magnetic/non-magnetic materials[19] or magnetic/magnetic materials[20, 21]. The dispersion relations were calculated by employing transfer matrix which depend on the thickness of the involved materials. However, in long-wavelength limit where the wavelength much longer than multilayer period, the calculation can be approached by applying an effective medium method[20, 22]. In this method, a multilayer configuration is represented by a unit medium having effective permittivity and effective permeability. According to Ref.[20], these effective parameters are calculated by employing Maxwell's continuity of the fields at the interface of the involved materials.

In the present paper we report in detail the study of polaritons in multilayer comprises of multiferroics/electric system. We show how the fraction of multiferroics affects both the bulk and surface polaritons. We provide numerical predictions of dispersion relations for multilayer comprise of $BaMnF_4/BaAl_2O_4$. Those materials are used since the model of multiferroic $BaMnF_4$ is frequently used in previous study[14–17] and there is a complete report about resonance frequencies of ferroelectric $BaAl_2O_4$[23]. We also discuss the properties of polaritons for both the transversal electric (TE) modes and transversal magnetic (TM) modes.

2. Geometry of the Multilayer

The geometry of the multilayer is illustrated in figure 1. The multilayer is semi-infinite which is comprise of multiferroics (MF) with the thickness d_1 and ferroelectric (E) with the thickness d_2. The surface of multilayer is lied in $x - y$ plane. Following the previous studies[16, 17], the multiferroics is ferroelectric-two sub-lattices antiferromagnet with canted spin.



(a) (b)

Figure 1. Geometry of the multilayer. (a) The multilayer is comprised of the ferroelectrics with the thickness d_1 and the multiferroics which is denoted by E with the thickness d_2. (b) The polarisation and the magnetisation of the multiferroics (MF). Polarisasion is directed in the y direction. The two-sublattice magnetisation is canted with canting angle θ yield weak ferrmognetism M.

The susceptibilites are obtained by examining the density energy of material in the equations of motion. In this process, we also obtain magnetic and electric resonance frequencies. The susceptibility components of this type of magnetoelectric multiferroic were formulated into two groups. The first group represents TE modes involving the electric component e_x and the magnetic components h_y and h_z. The susceptibility components for TE modes are[16]:

$$\chi^m_{yy} = \frac{2\omega_s \left(\omega_a \cos 2\theta + 2\omega_{me} \sin 2\theta + \omega_\circ \sin\theta\right)}{\left(\omega_M^2 - \omega^2\right)}, \tag{1}$$

$$\chi_{zz}^{m} = \frac{2\omega_s\left(\omega_{me}\sin 2\theta + \omega_o \sin\theta\right)}{\left(\omega_M^2 - \omega^2\right)}, \tag{2}$$

$$\chi_{yz}^{m} = -\chi_{zy}^{m} = \frac{i2\omega_s\omega\sin\theta}{\left(\omega_M^2 - \omega^2\right)}, \tag{3}$$

and

$$\chi_{x}^{e} = \epsilon_x^{\infty}\frac{\omega_{L,x}^2 - \omega^2}{\omega_{T,x}^2 - \omega^2} - 1 \tag{4}$$

where $\omega_M = \left(\omega_{afm}^2 \cos^2\theta + \Omega_{me,TE}^2 + \Omega_{o,TE}^2\right)^{1/2}$ represents magnetic resonance frequency in TE modes. The frequencies $\Omega_{me,TE} = \left[\omega_{me}\left(4\omega_{me}\cos\theta + \omega_a \sin 2\theta + 2\omega_{ex}\sin 2\theta\right)\right]^{1/2}$ is contributed from magnetoelectric interaction while $\Omega_{0,TE} = \left[\omega_0\left(\omega_0 - \omega_a\sin\theta + 4\omega_{me}\cos\theta\right)\right]^{1/2}$ is given by external magnetic field.

Here, the frequency $\omega_{ex} = \gamma\lambda\mu_0 M_s$ represent the frequency from exchange interaction while the frequency $\omega_a = \gamma K\mu_0 M_s$ is frequency which is contributed by anisotropy energy. Frequencies ω_{me} and ω_0 are defined as $\omega_{me} = \gamma\mu_0\alpha P M_s$ and $\omega_0 = \gamma\mu_0 H_0$. The frequency from sub-lattice magnetization is defined as $\omega_s = \gamma\mu_0 M_s$. The frequency $\omega_{afm} = \left[\omega_a\left(\omega_a + 2\omega_{ex}\right)\right]^{1/2}$ represents antiferromagnet frequency. In electric susceptibility, $\omega_{L,x}$ and $\omega_{T,x}$ represent the longitudinal and transverse phonon resonance frequency in x direction.

The TM mode which involves a magnetic component h_x and the electric components e_y and e_z has the magnetic and electric susceptibility components[16]

$$\chi_{xx}^{m} = \frac{2\omega_s\left(\omega_{el}^2 - \omega^2\right)\left(\omega_a\cos\theta + 2\omega_{me}\sin\theta\right)}{\left[\left(\omega_{el}^2 - \omega^2\right)\left(\omega_m^2 - \omega^2\right) - \Omega_s^4\right]}, \tag{5}$$

$$\chi_{y,2}^{e} = \frac{\eta/\epsilon_0\left(\omega_m^2 - \omega^2\right)}{\left[\left(\omega_{el}^2 - \omega^2\right)\left(\omega_m^2 - \omega^2\right) - \Omega_s^4\right]}, \tag{6}$$

$$\chi_{z}^{e} = \epsilon_z^{\infty}\frac{\omega_{L,z}^2 - \omega^2}{\omega_{T,z}^2 - \omega^2} - 1 \tag{7}$$

and also the magnetoelectric susceptibility in the form

$$\chi_{xy}^{me} = \chi_{yx}^{em} = \frac{4\alpha\left(\eta/\epsilon_0\right)M_s\omega_s\left(\omega_a\cos\theta + 2\omega_{me}\sin\theta\right)\cos 2\theta}{c\left[\left(\omega_{el}^2 - \omega^2\right)\left(\omega_m^2 - \omega^2\right) - \Omega_s^4\right]} \tag{8}$$

where frequency $\omega_m = \left(\omega_{afm,TM}^2 + \Omega_{me,TM}^2 + \Omega_{0,TM}^2 +\right)^{1/2}$ is defined as magnetic resonance frequency and $\omega_{el} = \left(\frac{\alpha_1}{\epsilon_0} + P_0\frac{\alpha_2}{\epsilon_0^2}\right)\eta$ represents the electric frequency. Here, η represents the inverse of the multiferroics' phonon mass and the frequency Ω_s in magnetoelectric susceptibility is defined as $\Omega_s = \left[8\frac{\eta}{\epsilon_0}\frac{\alpha^2 M_s^2}{c^2}\omega_s\left(\omega_a\cos^2\theta + 2\omega_{ex}\sin^2\theta\right)\right]^{1/4}$.

In this TM modes, $\omega_{afm,TM} = \left(\omega_a\left(\omega_a + 2\omega_{ex}\right)\cos^2\theta + 2\omega_{ex}\left(\omega_a + 2\omega_{ex}\right)\sin^2\theta\right)^{1/2}$ represents the antiferromagnetic resonance frequency in TM modes. The frequency from the magneto-electric interaction is represented by $\Omega_{me,TM} = \left[2\omega_{me}\left(4\omega_{me} - 2\omega ex\sin 2\theta + \omega_a\sin 2\theta\right)\right]^{1/2}$. The frequency $\Omega_{0,TM} = \left[\omega_0\left(\omega_0 - \omega_a\sin\theta - 4\omega_{ex}\sin\theta + 6\omega_{me}\cos\theta\right)\right]^{1/2}$ is contributed by the field H_0.

In this study, we simply use the electric susceptibilities for ferroelectric in the similar form as Eq.(5) and (7) with the form

$$\chi_{i,e}^{e} = \epsilon_{i,e}^{\infty}\frac{(\omega_{L,i}^{e})^2 - \omega^2}{(\omega_{T,i}^{e})^2 - \omega^2} - 1 \tag{9}$$

where $\omega^e_{T,i}$ represents a transverse optical lattice vibration frequency in the i direction and ω^e_L is cut off frequency of the forbidden band for waves propagation. Once all the magnetic and electric susceptibilites are defined, the permeabilities can easily be obtained through relation $\mu = I + \chi^m$ while the permittivities with the relation $\epsilon = I + \chi^e$.

In this calculation, we exclude intermixing interaction between ferroelectric and the electric part of the multiferroics as proposed in previous study[24]. Since this interaction will only slightly shift the electric resonance frequencies for both material, it will not significantly affect the feature of the dispersion relation curves.

3. Effective Medium

A multilayer system comprised of two or more materials can be approximated as a single material called effective medium[20, 25]. In this approximation, we assume the wavelength much longer than the thickness of each layer or periodicity of the multilayer. According to Ref.[20], the effective permittivity and permeability are calculated by considering the Maxwell's continuity at the interfaces.

A multilayer as illustrated in Fig.1(a) has the average of tangential electric displacement vector as

$$\bar{D}_x = \frac{d_1 D_1^x + d_2 D_2^x}{(d_1 + d_2)} = f_1 D_1^x + f_2 D_2^x \tag{10}$$

where f_1 and f_2 denote the volume fraction of the multiferroics and ferroelectrics. The displacement components D_1^x and D_2^x are the x component of the electric displacement vector of the multiferroics and ferroelectrics. In the next step, by considering the continuity of tangential electric fields, we have $E_1^x = E_2^x = \bar{E}_x$. Using constitutive equation $\vec{D} = \epsilon\vec{E}$ yield the relation in the form $\bar{D}_x = f_1\epsilon_1^x E_1^x + f_2\epsilon_2^x E_2^x = (f_1\epsilon_1^x + f_2\epsilon_2^x)\,\bar{E}_x$. Hence, the effective permittivity in the x component is written as

$$\epsilon_{eff}^{xx} = f_1\epsilon_1^x + f_2\epsilon_2^x. \tag{11}$$

Now, analyzing the average of the electric displacement, $\bar{D}_y = f_1 D_1^y + f_2 D_2^y$ using constitutive relation $D_1^y = \epsilon_1^{yy} E_1^y + \chi^{em} H_1^x$ for multiferroics and $D_2^y = \epsilon_2^{yy} E_2^y$ for ferroelectric , also by considering the continuity of the tangential component of magnetic field (H_x) and tangential component of the electric field (E_y), we obtain $\bar{D}_y = (f_1\epsilon_1^{yy} + f_2\epsilon_2^{yy})\,\bar{E}_y + f_1\chi^{em}\bar{H}_x$. Then, the y component of the effective permittivity can be defined as

$$\epsilon_{eff}^{yy} = f_1\epsilon_1^{yy} + f_2\epsilon_2^{yy} \tag{12}$$

and the effective magnetoelectric susceptibility as

$$\chi_{eff}^{em} = f_1\chi^{em}. \tag{13}$$

The z component of permittivity is derived by analyzing the average of the electric component $\bar{E}_z = f_1 E_1^z + f_2 E_2^z$ and the continuity of the normal component of electric displacement (D_z), result in the form

$$\epsilon_{eff}^{zz} = \frac{\epsilon_1^{zz}\epsilon_2^{zz}}{(f_1\epsilon_2^{zz} + f_2\epsilon_1^{zz})}. \tag{14}$$

If the same procedure above is performed to the magnetic induction field $\vec{B}$ and magnetic field $\vec{H}$, the effective permeability components are obtained in the form of

$$\mu_{eff}^{xx} = f_1\mu_1^{xx} + f_2\mu_2^{xx}, \tag{15}$$

$$\mu_{eff}^{yy} = \left\{ f_1\mu_1^{yy} + f_2\mu_2^{yy} + f_1^2 \frac{\mu_2^{zz}\mu_1^{yz}\mu_1^{zy}}{\left[f_2\left(\mu_1^{zz}\right)^2 + f_1\mu_1^{zz}\mu_2^{zz}\right]} - f_1\frac{\mu_1^{yz}\mu_1^{zy}}{\mu_1^{zz}} \right\}, \tag{16}$$

$$\mu_{eff}^{zz} = \left(\frac{\mu_1^{zz}\mu_2^{zz}}{f_1\mu_2^{zz} + f_2\mu_1^{zz}} \right) \tag{17}$$

and

$$\mu_{eff}^{yz} = \mu_{eff}^{zy} = f_1\frac{\mu_2^{zz}\mu_1^{yz}}{\left(f_1\mu_2^{zz} + f_2\mu_1^{zz}\right)}. \tag{18}$$

4. Theory of Polariton in MF/FE Multilayer

The behavior of electromagnetic waves propagation is illustrated by dispersion relation which is obtained by solving the Maxwell equations using the appropriate electromagnetic waves such as

$$\vec{\psi} = (\psi_x, \psi_y, \psi_z)\exp\left[i\left(k_y y + k_z z - \omega t\right)\right]. \tag{19}$$

Using configuration as illustrated in Fig.1(b), The Maxwell equations result in two groups of coupled equations. The first group which involves the H_y, H_z and E_x is classified as transverse electric (TE) mode. The second group is transverse magnetic (TM) mode which involves the fields components H_x, E_y and E_z.

Using electromagnetic waves as in Eq.(19) into Maxwell equations, the matrix equation for the TE modes which represent the bulk modes is

$$\begin{pmatrix} -k_z & k_y & \epsilon_{eff}^{xx}\frac{\omega}{c} \\ -\mu_{eff}^{yy}\frac{\omega}{c} & \mu_{eff}^{yz}\frac{\omega}{c} & k_z \\ \mu_{eff}^{zy}\frac{\omega}{c} & \mu_{eff}^{zz}\frac{\omega}{c} & k_y \end{pmatrix} \begin{pmatrix} H_y \\ H_z \\ E_x \end{pmatrix} = 0. \tag{20}$$

The solution requires the determinant matrix equal to zero which results in the form of bulk polariton as

$$k_y^2 = \epsilon_{eff}^{xx}\frac{\left[\mu_{eff}^{yy}\mu_{eff}^{zz} + \left(\mu_{eff}^{yz}\right)^2\right]}{\mu_{eff}^{yy}}\left(\frac{\omega}{c}\right)^2. \tag{21}$$

The TE surface modes are derived by assuming that the electromagnetic waves are decaying exponentially as they go into the materials. Hence, the appropriate solutions of the surface modes are

$$\vec{E} \sim e^{\beta z}e^{i(k_y y - \omega t)} \qquad \text{for } z < 0 \tag{22}$$

$$\vec{E} \sim e^{-\beta_o z}e^{i(k_y y - \omega t)} \qquad \text{for } z > 0 \tag{23}$$

where β and β_0 represent attenuation constant for multilayer and vacuum which define as

$$\mu_{eff}^{zz}\beta^2 = \mu_{eff}^{yy}k_y^2 - \epsilon_{eff}^{xx}\left[\mu_{eff}^{yy}\mu_{eff}^{zz} + \left(\mu_{eff}^{yz}\right)^2\right]\frac{\omega^2}{c^2}, \tag{24}$$

$$\beta_0^2 = k_y^2 - \frac{\omega^2}{c^2}. \tag{25}$$

The TE dispersion relation for the surface polaritons is derived by applying the continuity of tangential $\vec{H}$ (H_y) and the continuity of normal $\vec{B}$ (B_z) which result in the form

$$\mu_{eff}^{zz}\beta + \beta_0\left[\mu_{eff}^{yy}\mu_{eff}^{zz} + \left(\mu_{eff}^{yz}\right)^2\right] + i\mu_{eff}^{yz}k_y = 0. \tag{26}$$

Since the dispersion relation contains odd order of k_y, then surface polariton is non-reciprocal, with $\omega(k_y) \neq \omega(-k_y)$. Hence the non-reciprocality which was found in a pure PML-type multiferroics as reported in Ref.([16]) is still observable in this multilayer geometry.

The TM modes for bulk polaritons which is involving field components H_x, E_y and E_z are represented by a matrix equation resulted from the Maxwell equations as

$$\begin{pmatrix} \mu_{eff}^{xx}\frac{\omega}{c} & \left(k_z + \chi_{eff}^{me}\frac{\omega}{c}\right) & -k_y \\ \left(k_z + \chi_{eff}^{me}\frac{\omega}{c}\right) & \epsilon_{eff}^{yy}\frac{\omega}{c} & 0 \\ k_y & 0 & -\epsilon_{eff}^{zz} \end{pmatrix}\begin{pmatrix} H_x \\ E_y \\ E_z \end{pmatrix} = 0 \tag{27}$$

which result in the dispersion relation for bulk modes as

$$\epsilon_{eff}^{yy}k_y^2 = \left[\epsilon_{eff}^{yy}\mu_{eff}^{xx} - \left(\chi_{eff}^{me}\right)^2\right]\epsilon_{eff}^{zz}\frac{\omega^2}{c^2}. \tag{28}$$

The TM surface polaritons are calculated by assuming the solution is of the form

$$\vec{H} \propto e^{\beta z}e^{i(k_y y-\omega t)}. \tag{29}$$

where the attenuation constant β is defined by an equation such as

$$\epsilon_{eff}^{zz}\left(\beta + i\chi_{eff}^{me}\frac{\omega}{c}\right)^2 = \epsilon_{eff}^{yy}\left(k_y^2 - \mu_{eff}^{xx}\epsilon_{eff}^{zz}\frac{\omega^2}{c^2}\right). \tag{30}$$

The equation above illustrates that the attenuation constant β is complex. It means that the surface polariton modes are leaky modes, where there is an amount of energy leaking into the environment. Performing the similar procedure as in TE modes by analyzing the continuity of the fields (continuity of tangential $\vec{H}$, tangential $\vec{E}$ and normal $\vec{D}$) at the surface, results in

$$\left(\beta + \frac{i\omega}{c}\chi_{eff}^{me}\right) + \epsilon_{eff}^{yy}\beta_0 = 0. \tag{31}$$

The dispersion relation Eq.(31) reflects that the TM surface modes is reciprocal, $\omega(k_y) = \omega(-k_y)$.

5. Application to Multilayer $BaMnF_4/BaAl_2O_4$

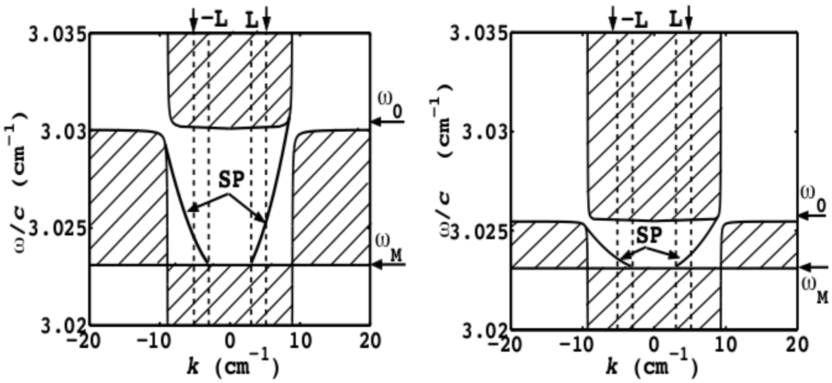
In this section, the previous theory is applied to the multilayer comprises of multiferroic $BaMnF_4$ and ferroelectric $BaAl_2O_4$. For $BaMnF_4$, some values of the variables are taken from the references such as: canting angle $\theta = 3$ mrad[26], polarisation $P_\circ = 0,115$ C/m^2[27], inverse phonon mass $f = 3 \times 10^{14}$ A^{2}s^{2}kg^{-1}m^{-3}[14], frequency $\omega_{ey} = 41$ cm^{-1}[28], $\omega_{ez} = 33.7$ cm^{-1}[14] and permittivity $\epsilon_x = 8.2$[29]. The others are estimated as in Ref.[16]. Sub-lattice magnetization $M_s = 2,4333 \times 10^5$ Am^{-1} is estimated using $M_c = 2M_s\sin\theta$ where weak ferromagnetism $M_c = 1460$ Am^{-1}[30]. An exchange constant $\lambda = 163{,}54$ is calculated using $H_E = \lambda\mu_0 M_s$ with $H_E = 50$ T[31]. An anisotropy constant $K = 0.337$ is approximated using $\omega_r^2 = \gamma^2\mu_0^2K(K-2\lambda)M_s^2$ with $\omega_r = 3$ cm^{-1}[28]. The calculated magnetoelectric coupling α is obtained using relation $\tan(2\theta) = \frac{4\alpha P}{K+2\lambda}$ as in Ref.[16]. The values of variables for $BaAl_2O_4$ are taken from Ref.[23] such as $\epsilon_\perp^\infty = 3.12$, $\epsilon_{//}^\infty = 3.13$, $\epsilon_\perp^0 = 9.82$, $\epsilon_{//}^0 = 11.09$, frequency $\omega_{t,\perp} = 63$ cm^{-1} and $\omega_{t,//} = 63$ cm^{-1}.

The solutions of the TE dispersion relation for bulk in Eq.(21) and surface modes in Eq.(26) were obtained numerically using root finding technique. For the volume fraction of multiferroics $f_1 = 0.75$, the solutions of the surface dispersion relation in TE modes was found at around the magnetic resonance frequency of $BaMnF_4$ ($\omega_M \approx 3.023$ cm^{-1}) which is illustrated in Fig.2(a). In that figure, the surface modes are represented by thick lines which are denoted 'SP' while the bulk bands are represented by shaded regions.

Since around the magnetic frequency ω_M, permittivity component ϵ_{xx} of multiferroic $BaMnF_4$ is constant at the value 8.2 while the permittivity ϵ_{xx} of ferroelectric $BaAl_2O_4$ is also constant around the value 3.12, hence the value of the effective permittivity ϵ_{eff}^{xx} can be approximated as constant. Then, based on Eq.(21), the bulk bands comprise of only two bands which are separated by an area where the surface modes may exist. This region is limited by magnetic resonant frequency ω_M and the zeros frequency ω_O of the dispersion relation for bulk polaritons (see Fig.2(a)).

Since the magnetic resonance frequency is only contributed by multiferroic $BaMnF_4$, then the bulk equation in Eq.(21) can be brought into

$$k_y^2 \propto (f_1 + f_2\mu_{yy})\,\mu_{zz} - f_2\mu_{yz}^2 \tag{32}$$



(a) (b)

Figure 2. Dispersion relation of Multilayer with various volume fraction of Multiferroics. In (a) with the fraction $f_1 = 0.75$ and (b) with $f_1 = 0.25$. Shaded region is the area for bulk polaritons. Surface modes are represented by thick lines which are denoted by 'SP"'. The dashed lines which are denoted by L represent Lightlines. The ATR line are illustrated by dashed lines with the downward arrows.

where the denominators of μ_{yy}, μ_{zz} and μ_{yz} have the same resonance frequency. Hence, the volume fraction of the multiferroic layer do not affect the magnetic resonance frequency of the multilayer as illustrated in Fig.2(a)-2(b).

The zeros frequency ω_0 can be calculated by solving the condition

$$\left[\mu_{eff}^{yy}\mu_{eff}^{zz} + \left(\mu_{eff}^{yz}\right)^2\right] = 0 \tag{33}$$

which can be brought into cubic equation $\omega^4 + B\omega^2 + C = 0$ where B and C were defined as $B = -\left(f_1\omega_M^2 + f_2\omega_{oy}^2 + \omega_{oz}^2 - f_2\omega_s^2\sin^2\theta\right)$ and $C = \left(f_1\omega_M^2 + f_2\omega_{oy}^2\right)\omega_{oz}^2$. The frequencies ω_{oy} and ω_{oz} represent zero frequency of the permeability components μ_{yy} and μ_{zz}. The solution of cubic equation is then $\omega_0 = \left[\frac{-B+\sqrt{B^2-4C}}{2}\right]^{1/2}$.

It was found that by reducing the volume fraction of the multiferroics $BaMnF_4$, the zeros frequency of bulk polariton will decrease as illustrated in Fig.2(a)-2(b). The zeros frequency drop from 3.031 cm^{-1} to 3.026 cm^{-1} when the volume fraction was reduced from 75% to 25%. This zeros frequency reduction result in the decrement of the region where the surface modes may exist, since the magnetic resonance frequency is independent of the volume fraction. Hence, it also shifts the surface modes curves.

Since the attenuation total reflection (ATR) has been successfully used to identify the surface polaritons in antiferromagnet FeF_2[13, 32], in this report we also calculate the ATR reflectance to verify the polaritons generation. ATR reflectance is reflection of the incident waves in ATR system which is took place at the material sample. In this method, the existence of surface modes is represented by sharp dips while the bulk polaritons are illustrated with shallow dips of the ATR reflectance spectrum. The ATR reflectivity is calculated by considering the reflectance at the multilayer surface and also at the bottom of high index prism which results in the formula[33]

$$R = \left| \frac{k_z\left(1 + re^{-2\beta_0 d}\right) - i\beta_0\left(1 - re^{-2\beta_0 d}\right)}{k_z\left(1 + re^{-2\beta_0 d}\right) + i\beta_0\left(1 - re^{-2\beta_0 d}\right)} \right| \tag{34}$$

where k_z is propagation normal to the surface of the multilayer. Here, $r = \frac{\beta_0 - \zeta}{\beta_0 + \zeta}$ represents reflectivity at the surface of multilayer with $\zeta = \frac{\left(\mu^{zz}_{eff}\beta - i\mu^{yz}_{eff}k_y\right)}{\left[\mu^{yy}_{eff}\mu^{zz}_{eff} + \left(\mu^{yz}_{eff}\right)^2\right]}$.

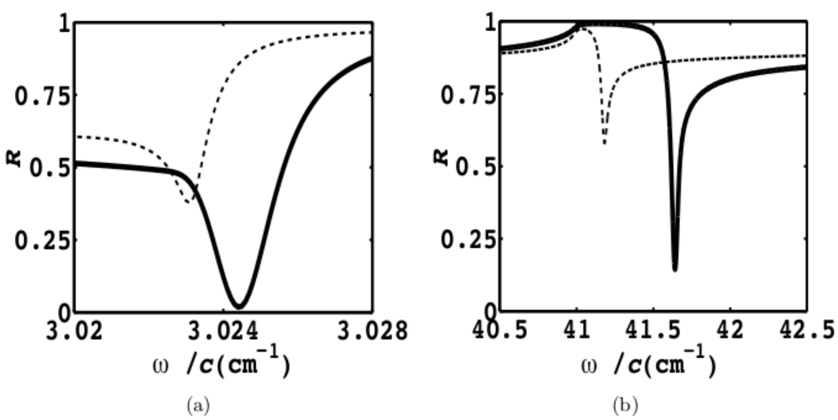
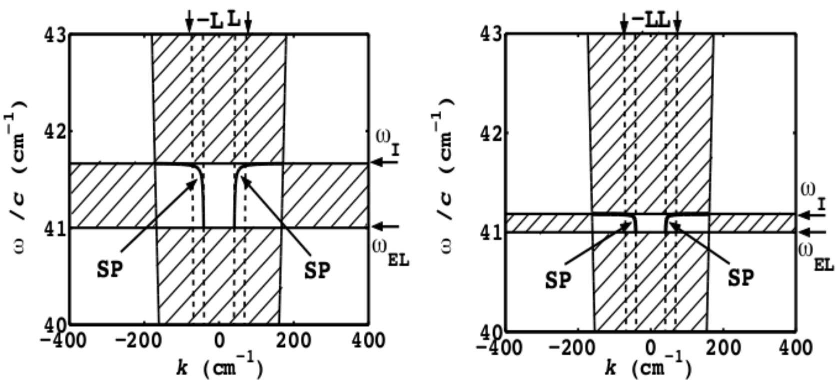


Figure 3. Calculated ATR Reflectivity with incident angle 30^0. In (a), The ATR spectroscopy for TE mode, (b) The ATR for TM mode. The dashed line represents the volume fraction $f_1 = 0.25$, while the solid thick lines is for$f_1 = 0.75$.

In figure 2(a), the dashed lines denoted by arrow '↓' represent ATR line with incident angle at 30^0. This ATR line intersects the surface modes at around 3.025 cm^{-1}, agree with the sharp dip of the solid line in the calculated ATR reflectance at Fig.3(a). The shift of surface modes curves due to the reduction of volume fraction of the multiferroics to $f_1 = 0.25$ can also be observed as illustrated as a dashed line.

The solutions of the dispersion relation in TM modes for bulk in Eq.(28) and surface in Eq.(31) are illustrated in Fig.4(a). Here, the volume fraction of the multiferroics $BaMnF_4$ is 0.75. The shape of polaritons around the phonon frequency is also similar to the previous report for the pure $BaMnF_4$[16]. However, in this report, the 'window' where the surface modes may exist is thinner for the volume fraction $f_1 = 0.75$ than the 'window' in the pure multiferroics.



(a) (b)

Figure 4. Surface polaritons for TM modes. (a) Surface modes with volume fraction of multiferroics $f_1 = 0.75$. (b) with volume fraction $f_1 = 0.25$. The bulk bands are represented by the shaded regions, while the surface modes are represented by thick lines which denote by 'SP'.

The existence of both surface and bulk modes in this modes is confirmed by the calculated ATR spectrum as illustrated in Fig.3(b). The ATR reflectance for TM mode is also given by formula in Eq.(34) with the reflectivity r is now given as $r = \frac{\epsilon_{eff}^{yy}\beta_0 - \beta}{\epsilon_{eff}^{yy}\beta_0 - \beta}$.

When the volume fraction of the multiferroics is decreased to 0.25, the 'window' where the surface modes are present become very narrow as illustrated at Fig.4(b). This decrement is resulted from the reduction of the zero frequency ω_I while the shifted resonance frequency ω_{EL} is not affected by the reduction of volume fraction of the multiferroics. The zero frequency ω_I obey the condition

$$\epsilon_{eff}^{yy}\mu_{eff}^{xx} - \left(\chi_{eff}^{me}\right)^2 = 0. \tag{35}$$

Near the phonon resonance frequency, the permeability $\mu_{eff}^{xx} \cong 1$ while the value $\epsilon_{eff}^{yy} >> \chi_{eff}^{me}$. Hence the Eq.(35) can be approximated by

$$\epsilon_{eff}^{yy} = f_1\epsilon_1^{yy} + f_2\epsilon_2^{yy} = 0. \tag{36}$$

Then, by considering that permittivity $BaAl_2O_4$ around the phonon frequency of the multiferroics is approximately 17, the Eq.(36) above can be brought into a cubic equation as

$$A\omega^4 + B\omega^2 + C = 0. \tag{37}$$

Here, $A = 1 + \kappa$ with $\kappa = \frac{f_2}{f_1}\frac{\epsilon_2^{yy}}{\epsilon_{yy,1}^{\infty}} \approx \frac{f_2}{f_1}\frac{17}{\epsilon_{yy,1}^{\infty}}$. The parameters B and C are defined as $B = -(1+\kappa)(\omega_{mag}^2 + \omega_{ey}^2) - \omega_f^2$ and $C = (1+\kappa)(\omega_{mag}^2\omega_{ey}^2 - \Omega_c^4) + \omega_f^2\omega_{mag}^2$.
Hence, the zeros frequency ω_I can be solved easily as

$$\omega_I = \left\{\frac{-B + \sqrt{B^2 - 4AC}}{2A}\right\}^{1/2}. \tag{38}$$

The decrease of the region where the surface modes exist will suppress the surface modes curves. As a consequence, the surface modes curves become shorter as the volume fraction of multiferroics is decreased. As it is illustrated in Fig.3(b), the calculated ATR reflectance is also showing a downward shift in surface modes curves when the volume fraction is decreased.

6. Conclusion

We show how the dispersion relations of multiferroic are modified in multilayer configuration for both TE and TM modes, even though the reciprocity or non-reciprocity of the multiferroics does not change by arranging multiferroics into a multilayer configuration with electric material. The dispersion relations of the multilayer in both TE and TM modes are affected by the volume fraction of the multiferroics. The region where the surface mode exist is narrow when the volume fraction of multiferroic in multilayer system is small (around 25%). In multilayer with the volume fraction of multiferroic is relatively big (around 75%), the region for surface modes is wide.

Acknowledgments

We wish to acknowledge the support of Ditlitabmas Ditjen Dikti Kemendikbud through DIPA Diponegoro University.

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