Constructing Irislet: a New Wavelet Type which Matched for Iris Image Characteristics

R. Rizal Isnanto
Computer Engineering Department
Diponegoro University
Semarang, Indonesia
rizal_isnanto@undip.ac.id

Kodrat Iman Satoto
Computer Engineering Department
Diponegoro University
Semarang, Indonesia
kodratis@yahoo.com

Ike Pertiwi Windasari
Computer Engineering Department
Diponegoro University
Semarang, Indonesia
ikepertiwi@gmail.com

***Abstract* – Iris has a unique pattern that can be used in biometric recognition. To extract the features of the iris, it can be done based on the textural characteristics of the iris pattern. One method is a texture-based feature extraction using wavelet. To construct a wavelet type which matched for a signal, in this case two-dimensional signal from the iris image, the necessary steps are quite complex. In this research, all stages of wavelet design are carried out, beginning from iris image data acquisition up to the finding of the new wavelet, which will then be referred to as irislet. There are 19 (nineteen) steps in the design of this wavelet. To do all the stages, several basic concepts are required: convolution, circular Hough transform, conversion into unwrapped polar image form, determining the profile of the 1-D line images, signal averaging, concept of Daubechies wavelet basis, calculating signal energy, least squares method, how to construct scaling and wavelet functions, as well as the cascade algorithm. The test results showed that the recognition implementation irislet shows recognition rate is 100% correct.**

Keywords – irislet; scaling function; wavelet function; least squares method; cascade algorithm.

I. INTRODUCTION

A. Background

Man as an individual, has a unique and distinctive characteristics. These characteristics can be used as recognition or identification of a person. This is known as biometric recognition. Iris is the part of the eye surrounding the pupil. Although iris has a relatively narrow compared to the broad human body, iris has a unique pattern, different in each individual, and the pattern will remain stable. Based on this, the iris can be used as the basis for biometric recognition.

Many algorithms have been observed for iris feature extraction, such as Principal Component Analysis (PCA), Independent Component Analysis (ICA), Gabor-wavelet algorithm [1, 2], characterizing Key Local Variation, Laplace Pyramid, Gray Level Co-ocurrence Matrix (GLCM) [3] and others. Wavelet as one method to analyze the texture is used to extract the features of the iris, but still uses standard wavelet types, for example: Haar, Daubechies, Coiflet, Symlet, and Biorthogonal. In this research, a new type of wavelet will be developed, which will be referred to as **irislet**, and match the characteristics of the iris image.

B. Research Objective

The objective of the research is to construct a wavelet types are suitable for a signal, in this case the signal from the 2-dimensional iris image. In this research, all stages of design of wavelet are carried out, beginning from eye image data acquisition until obtaining a new wavelet.

II. BASIS OF THEORY

A. Iris

Iris can serve as the basis for biometric systems. Each iris has a texture that is very detailed and unique to each person and remain stable for decades. This part of eye can not be altered through surgery without causing any damage to eyesight. Figure1 shows the anatomy of the eye, and examples of human iris [4].

The advantages of using the iris for reliable identification system are [5] as follows.

1. Iris is insulated and shielded from the outside environment.
2. In the iris, it is not possible to do some surgeries without causing defects in the eye.
3. Iris has a physiological response to light, which allows testing of the natural use of the possibility of fraud and faked eye lenses and so forth.

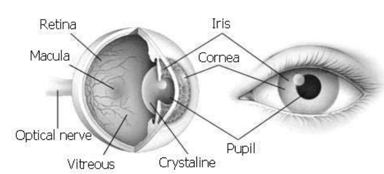


Figure 1. Anatomy of eyes and example of iris region

B. Wavelet

Wavelet is a mathematical function that meet certain requirements which is able to perform the decomposition of a function [6]. Hierarchically, to represent data or other functions. wavelet can be used to describe a model or the original image into a mathematical function regardless of the form of the model in the form of an image, a curve or a plane. Wavelet is a function that transforms the signal from the time to the frequency and scale.

Wavelet is most appropriately used in image processing because not much information is lost during the process of image reconstruction. Wavelet is a base. Wavelet bases derived from a scaling function. Scaling function has properties that can be developed from a basic function that has been dilated, translated and scaled. This function is derived from the dilation equation, which is considered as the basis of the theory of wavelets.

III. MATERIALS AND METHODS

Materials/research material taken from 2 (two) sources, namely: eye image taken from digital camera assemblies Irdosoft 4.0 with a resolution of 5 megapixels and the image of eye obtained from CASIA database [7] - "Portions of the research in this paper use the CASIA-IrisV1 collected by the Chinese Academy of Sciences' Institute of Automation (CASIA) "

Some of the requirements to iris samples are: the normal eye of adult respondents; healthy eye condition, no defects, no such abnormality that is stated Theory Foundation; should be taken from the respondents were not wearing glasses (must be less than minus 3), and not taken postsurgery.

The implementation of the program is supported by the Matlab version 7.10.0 (R2010a) [8] with the toolbox used include: Image Processing, Wavelet [9], and Optimization Toolbox.

Before implementing several steps for constructing a new wavelet, a preprocessing should be conducted. In order to segment the iris object to other parts of eye (i.e. especially pupil and sclera), a circular Hough transform is applied [10].

In general, it can be explained that the process of designing a suitable wavelet for 2-D iris signal partially follow the steps as done by Guido [11] in developing spikelet characteristics based on the existing signal and by Gupta, *et al.* [12] in designing a new approach for estimating wavelet matched to signal. This process can be illustrated in the diagram shown in Figure 2, followed by a detailed description of the orderly series: A, B, and C.

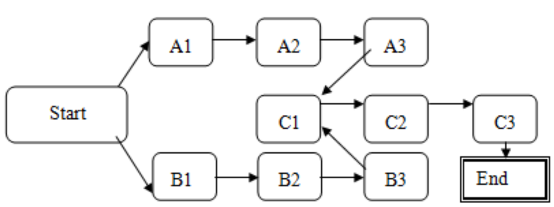


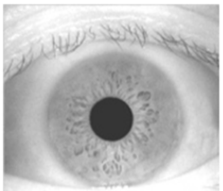
Figure 2. Stage series to construct a new wavelet

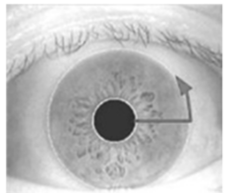
Here are 19 steps developing the new wavelet.

1) Selecting the image of the eye as input image.
2) Performing the segmentation process to take only part of the iris and dispose of any other part.
3) Converting the coordinates in the form of unwrapped iris.
4) Selecting the rows in the image, then the line profiles of one-dimensional signals are determined.
5) Repeating 4th step for the components of the column to obtain the line profiles.
6) Taking the base signal candidates from 4th and 5th steps seemed to be the shortest signal that dominantly form the the line profile.
7) Averaging the signals. The mean signal is used as the basis for calculating the signal type that designed short wave.
8) Developing equations for the filter coefficients of Daubechies (Step A1) and throwing orthogonality requirements (Step A2). At this stage 3 (three) equations with 4 (four) of unknown value h_k $(0 \leq k \leq 3)$ will be obtained (Step A3).
9) Convolving the result item 7 with generic signal support $n = 4$ is unknown, ie. h_3 ... h_0 (Step B1).
10) Perform the downsampling process by a factor of 2 over the convolution result item point 9.
11) Signal energy of point 10 is maximized by deriving the partial signal with each coefficient h_3 ... h_0, and the result equated to zero (Step B2).
12) Item (11) generates 4 equations with 4 unknown variables (Step B3).
13) Item 8 and 12 are combined to produce seven equations with 4 unknown variables (Step C1).
14) Overdetermined linear equations in point 13 then are solved using the least squares method to obtain the value of h_3 ... h_0 which forms a low pass filter (LPF) (Step C2).
15) From the values derived from point 14 is then determined high-pass filter reflection (mirror highpass filter) $g_3...g_0$ (Step C3).
16) Determining the scaling function $\phi(n)$ recursively using the dilation equation, as well as the point value of which $(\phi(n/2))$ is determined.
17) Determining the wavelet function $\psi(n)$ and also $\psi(n/2)$.
18) Plotting the scaling function and wavelet functions in graphs.
19) Using the cascade algorithm up to iteration-20 to produce smoother scaling function and wavelet functions.

RESULTS AND DISCUSSION

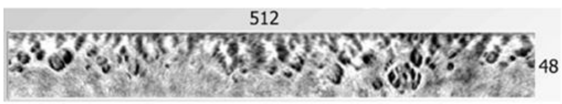
In this step, the searching of an appropriate wavelet bases was implemented, so that the certain signals can be modeled suitable for signal processing purposes using the wavelet [13]. For example, from the original iris image (Figure 3(a)), followed by localization of the iris area (Figure 3(b)), then the process of converting the image into the unwrapped form, as shown in Figure 3 (c) and improving the quality of the image as shown in Figure 3 (d).





(a) original eye image (b) iris image localization

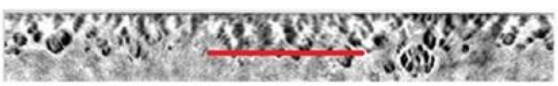
(c) iris image in wrapped-form



(d) quality-enhanced iris image in wrapped-form

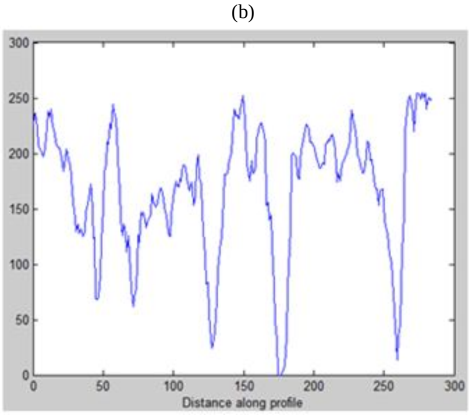
Figure 3. Processing of one iris image for signal characterictics testing

Meanwhile, the distribution of pixel values can be shown on the graph shown in Figure 4. Horizontal axis of the figure shows the relative positions of pixels on a line is observed, while the vertical axis shows the number of pixels of the relative position, with the horizontal axis length of about 300 pixels.



(a) example of red horizontal line to be observed

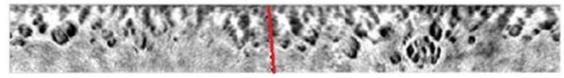
(b)



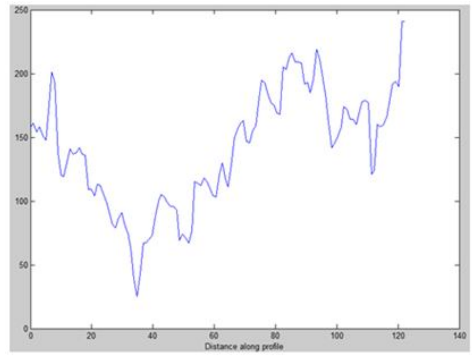
(b) one-dimensional signal characteristics of point (a)

Figure 4. Pixel distribution of 300-pixel length horizontal line

Whereas Figure 5 shows the characteristics of the distribution of pixels on the vertical line along 140 pixels.



(a) example of red vertical line to be observed



(b) one-dimensional signal characteristics of point (a)

Figure 5. Pixel distribution of 140-pixel length horizontal line

Using Daubechies terms, the following equation is obtained.

$$\begin{cases} -1h_3 + 1h_2 - 1h_1 + 1h_0 = 0 \\ -3h_3 + 2h_2 - 1h_1 + 0h_0 = 0 \\ 1h_3 + 1h_2 + 1h_1 + 1h_0 = 2 \end{cases} \qquad (1)$$

Average signal of the one-dimensional image of the iris is determined by taking the distribution of the profile of a line on the iris image that describes the relationship between the position of pixels with intensity values. Profiles are taken in two directions, both horizontal and vertical directions.

Meanwhile, Figure 6 shows some examples of candidates of base signal that is used to make the wavelet function. From the base signal candidates, then the base signal is determined by averaging the candidate signals, as shown in Figure 7.

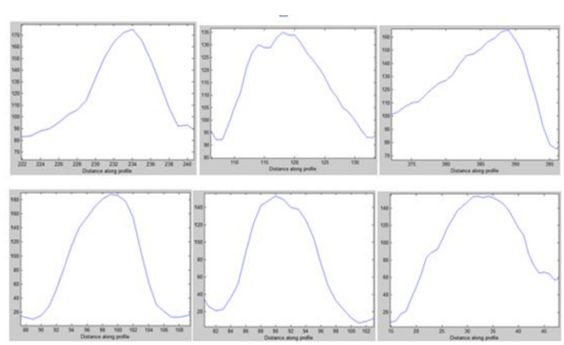
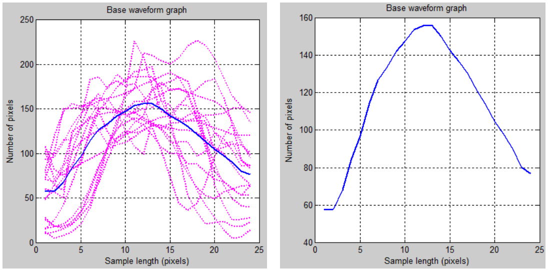


Figure 6. Some signal candidates for wavelet basis function



(a) (b)

Figure 7. Average signal of wavelet basis
(a) constructing signals waveform
(b) basis signal waveform for wavelet function

The values that show the relationship between the length of the sample (in pixels) by the number of pixels on the long position mentioned samples, can be shown in Table 1. To simplify the calculation, the intensity values in Table 1 are normalized by dividing by the greatest number, which is 156.3529 so that its value is in the range $0 \le$ intensity value ≤ 1. Normalization produces normalized values as shown in Table 2.

TABLE 1. INTENSITY VALUES OF PIXELS IN BASE SIGNAL

Pixel's Position	Intensity value	Pixel's Position	Intensity value	Pixel's Position	Intensity value	Pixel's Position	Intensity value
0	57,3529	6	126,8824	12	156,3529	18	113,9412
1	57,6471	7	133,7059	13	150,1765	19	104,7647
2	67,9412	8	142,4118	14	142,4706	20	97,5882
3	83,9412	9	147,5294	15	137,0588	21	90,3529
4	96,9412	10	153,8824	16	130,4706	22	80,1765
5	114,5294	11	155,8235	17	121,3529	23	76,7059

TABLE 2. NORMALIZED INTENSITY VALUES OF PIXELS IN BASE SIGNAL

Pixel's Position	Intensity value	Pixel's Position	Intensity value	Pixel's Position	Intensity value	Pixel's Position	Intensity value
0	0,3668	6	0,8115	12	1,0000	18	0,7287
1	0,3687	7	0,8552	13	0,9605	19	0,6701
2	0,4345	8	0,9108	14	0,9112	20	0,6242
3	0,5369	9	0,9436	15	0,8766	21	0,5779
4	0,6200	10	0,9842	16	0,8347	22	0,5128
5	0,7325	11	0,9966	17	0,7761	23	0,4906

After that, the signals in Table 2 are convolved with generic signal with support $n = 4$ with h_0... h_3 unknown. In this case the signals are convolved with $[\,h_3 \;\; h_2 \;\; h_1 \;\; h_0\,]$, and the herebelow is the calculation.

$$[0{,}3668\ 0{,}3687\ 0{,}4345\ 0{,}5369 \ldots 0{,}4906] * [h_3\ h_2\ h_1\ h_0]$$

$$= 0{,}3668h3 + 0{,}3687h3 + 0{,}3668h2 + 0{,}4345h3 + 0{,}3687h2 + 0{,}3688h1 + 0{,}5639h3 + 0{,}4345h2 + 0{,}3687h1 + 0{,}3688h0 + \ldots$$

(2)

followed by down-sampling by a factor of 2. The resulting signal is a one-dimensional projections of the iris image, called a vector $\vec{u}$ on the subspace V.

Each value of the convolution in Equation (2) is squared to determine the signal energy. After that, the signal energy is maximized by partially differentiating the signal with each coefficient h_3 .... h_0, and the result equated to zero.

$$\frac{\partial E(\overrightarrow{P_v u})}{\partial h_3} = 0,\quad \frac{\partial E(\overrightarrow{P_v u})}{\partial h_2} = 0,\quad \frac{\partial E(\overrightarrow{P_v u})}{\partial h_1} = 0,\quad \frac{\partial E(\overrightarrow{P_v u})}{\partial h_0} = 0 \tag{3}$$

$$\frac{\partial E(\overrightarrow{P_v u})}{\partial h_3} = -6{,}8748h_3 + 6{,}8645h_2 - 6{,}8748h_1 + 6{,}5373h_0 = 0$$

$$\frac{\partial E(\overrightarrow{P_v u})}{\partial h_2} = -6{,}8645h_3 + 6{,}8748h_2 - 6{,}8645h_1 + 6{,}6088h_0 = 0$$

$$\frac{\partial E(\overrightarrow{P_v u})}{\partial h_1} = -6{,}6088h_3 + 6{,}8645h_2 - 6{,}8748h_1 + 6{,}8645h_0 = 0$$

$$\frac{\partial E(\overrightarrow{P_v u})}{\partial h_0} = -6{,}5372h_3 + 6{,}6088h_2 - 6{,}8645h_1 + 6{,}8748h_0 = 0 \tag{4}$$

Equation (1) and equation (4) then combined, resulting in a system with $(n/2 + 1) + n = 3n/2 + 1 = 7$ linear equations with n = 4 unknown variables in the form **A.h** = **Y** as follows.

$$\begin{bmatrix} -6{,}8748 & 6{,}8645 & -6{,}6088 & 6{,}5372 \\ -6{,}8645 & 6{,}8748 & -6{,}8645 & 6{,}6088 \\ -6{,}6088 & 6{,}8645 & -6{,}8748 & 6{,}8645 \\ -6{,}5372 & 6{,}6088 & -6{,}8645 & 6{,}8748 \\ -1 & 1 & -1 & 1 \\ -3 & 2 & -1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} h_3 \\ h_2 \\ h_1 \\ h_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 2 \end{bmatrix} \tag{5}$$

The system then is solved using the least squares method. The problems here is how to solve the overdetermined linear equations are expressed in the following matrix [14].

$$\mathbf{A.x} = \mathbf{b} \tag{6}$$

where **A** is a matrix of size $m \times n$ with $m > n$. For b $\notin$ range (**A**) then there is no solution for **x**. The formulation of the least-squares method is:

$$\min \|Ax - b\| = \left(\sum_{i=1}^{m}\left(\sum_{j=1}^{n} a_{ij}x_j\right)^2\right)^{\frac{1}{2}} \tag{7}$$

The value of $r = A.x - b$ is called the residual or error, while x with least residual norm $||r||$ is called as least-squared solution. The solution of (5) is equivalent with the solution of:

$$\min \|A.x - b\|^2$$

which for getting the optimal value of x, first derivative with should be conducted from $||Ax - b||^2$ with respect to x, the value of the first derivative of the function is 0 (zero).

From the calculations using least-squares method, the cvalue $h_3..h_1$ can be obtain as follows.

$h_0 = 0{,}2742595$ $\quad$ $h_2 = 0{,}7189332$

$h_1 = 0{,}7583832$ $\quad$ $h_3 = 0{,}2341420$

The above values of h then constructs a low-pass filter. If defined as QMF (*quadrature mirror filters*) system, the digital filters defined by coefficients of $h_0...h_3$ have its filters' couple with coefficients $g_0...g_3$ which construct its mirror highpass filter, as stated by Hamming (1989). In relation with its frequency response, then it can be given as:

$$g_k = (-1)^k h_{M-k-1},$$

where M indicates the support length of its coefficients number, i.e. 4 (four). The followings are the coefficients values of g_0 ... g_3.

$g_0 = h_3 = 0{,}2341420$ $\quad$ $g_2 = h_1 = 0{,}7583832$

$g_1 = - h_2 = - 0{,}7189332$ $\quad$ $g_3 = - h_0 = - 0{,}2742595$

Scaling Function: Scaling function $\phi(x)$ can be obtained recursively using dilation equation:

$$\phi(n) = \sum_{k=0}^{M-1} h_k \phi(2n-k) \tag{8}$$

where support n = 4 coefficients, then:

$$\begin{cases} \phi(0) = h_0\phi(0) \\ \phi(1) = h_0\phi(2) + h_1\phi(1) + h_2\phi(0) \\ \phi(2) = h_1\phi(3) + h_2\phi(2) + h_3\phi(1) \\ \phi(3) = h_3\phi(3) \end{cases} \tag{9}$$

or it can be indicated as $M.T = T$, where

$$M = \begin{bmatrix} h_0 & 0 & 0 & 0 \\ h_2 & h_1 & h_0 & 0 \\ 0 & h_3 & h_2 & h_1 \\ 0 & 0 & 0 & h_3 \end{bmatrix} \text{ and } T = \begin{bmatrix} \phi(0) \\ \phi(1) \\ \phi(2) \\ \phi(3) \end{bmatrix} \tag{10}$$

Therefore, matrix T with scaling function value is an eigenvector of M for eigenvalue of 1. Using normalization condition $\sum_k \phi(k) = 1$, it can be obtained that

$$\begin{cases} (h_0 - 1)\phi(0) = 0 \\ h_2\phi(0) + (h_1 - 1)\phi(1) + h_0\phi(2) = 0 \\ h_3\phi(1) + (h_2 - 1)\phi(2) + h_1\phi(3) = 0 \\ (h_3 - 1)\phi(3) = 0 \\ \phi(0) + \phi(1) + \phi(2) + \phi(3) = 1 \end{cases} \tag{11}$$

Because of $h_0 ... h_3 \neq 0$, therefore

$$\begin{cases} \phi(0) = \phi(3) = 0 \\ (h_1 - 1)\phi(1) + h_0\phi(2) = 0 \\ h_3\phi(1) + (h_2 - 1)\phi(2) = 0 \\ \phi(0) + \phi(1) + \phi(2) + \phi(3) = 1 \end{cases} \tag{12}$$

By replacing the values of $h_3...h_0$ into Equation (12), we can get the following equations.

$$\begin{cases} -0{,}2416\phi(1) + 0{,}2743\phi(2) = 0 \\ 0{,}2341\phi(1) - 0{,}2811\phi(2) = 0 \\ \phi(1) + \phi(1) = 0 \end{cases}$$

for which, using least-squares method, it can be determined the solutions of the values of $\phi(1)$ and $\phi(2)$ as follows.

$$\phi(1) = 0{,}5386$$
$$\phi(2) = 0{,}4613$$

After that, based on Equation (8), equations for the midpoints which meet the condition of:

$$\phi(x/2) = \sum_{k=0}^{3} h_k \phi(x-k)$$

are

$$\begin{cases} \phi\left(\frac{1}{2}\right) = h_0\phi(1) = (0{,}2743)(0{,}5386) = 0{,}14774 \\ \phi\left(\frac{3}{2}\right) = h_1\phi(2) + h_2\phi(1) = 0{,}73705 \\ \phi\left(\frac{5}{2}\right) = h_3\phi(2) = (0{,}2341)(0{,}4613) = 0{,}10799 \end{cases}$$

Wavelet functions: Equations for wavelets function $\psi(n)$ can be derived from the equation:

$$\psi(n) = \sum_{k=0}^{3} g_k \, \phi(2n-k) \tag{13}$$

In accordance with the condition of Equation (8) that $\phi(x) \neq 0$ is true when $0 \leq x \leq 3$, therefore, according to the equation (13), we get

$$\begin{cases} \psi(0) = \psi(3) = 0 \\ \psi(1) = g_0\phi(2) + g_1\phi(1) = -0{,}27921 \\ \psi(2) = g_2\phi(2) + g_3\phi(1) = 0{,}22332 \end{cases}$$

and the equations for the midpoints which meet the condition of:

$$\psi(x/2) = \sum_{k=0}^{3} g_k \emptyset(x-k)$$

are

$$\begin{cases} \psi\left(\frac{1}{2}\right) = g_0\emptyset(1) = 0{,}12609 \\ \psi\left(\frac{3}{2}\right) = g_1\emptyset(2) + g_2\emptyset(1) = 0{,}07685 \\ \psi\left(\frac{5}{2}\right) = g_3\emptyset(2) = -0{,}12653 \end{cases}$$

Approximations to scaling functions and wavelet functions irislet at iteration from 1 to 20 using the cascade algorithm can be shown in Figure 8. Note that the bold lines in both figures indicate the final form of both scaling and wavelet functions.

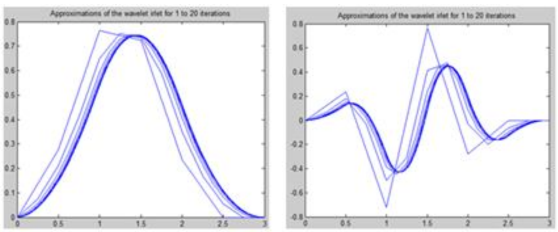


Figure 8. Scaling function (left) and wavelet function (right) of irislet at 20th iteration using cascade algorithm

With the values of scaling functions and wavelet functions described above, irislet has parameters h (scaling factor) and g (wavelet coefficients) as follows.

h_0 = 0,2742595	$g_0 = h_3$ = 0,2341420
h_1 = 0,7583832	$g_1 = - h_2$ = - 0,7189332
h_2 = 0,7189332	$g_2 = h_1$ = 0,7583832
h_3 = 0,2341420	$g_3 = - h_0$ = - 0,2742595

The properties of irislet as a new wavelet are that it is a non-orthogonal wavelet and the filter length is $N = 4$. While, testing on recognition rate of this new type of wavelet: irislet yield the following results. From 30 test images, irislet gives 100% recognition rate or there is no image that is recognized as 'false'.

V. CONCLUSIONS

From the research results and discussion, some conclusions can be taken. These are:

1. To construct a wavelet type which matched for a signal, in this case two-dimensional signal from the iris image, the necessary steps are quite complex. There are 19 (nineteen) steps in the design of this wavelet, beginning from iris image data acquisition up to the finding of the new wavelet, which will then be referred to as **irislet**.
2. As a new wavelet type, **irislet** has properties: non-orthogonal filter and has filter length 4.
3. To do all the stages of desingning new wavelet, several basic concepts are required: convolution, circular Hough transform, conversion into unwrapped polar image form, determining the profile of the 1-D line images, signal averaging, concept of Daubechies wavelet basis, calculating signal energy, least squares method, how to construct scaling and wavelet functions, as well as the cascade algorithm.
4. Irislet filter produces a good recognition rate, i.e. 100% as applying Haar filter.

ACKNOWLEDGMENT

R.R. Isnanto wishes to acknowledge the assistance or encouragement from colleagues, they are: Mr. Rukun Santoso and Mr. Achmad Hidayatno who had given some interesting views for insight understanding into wavelets.

REFERENCES

[1] Wijayanto, W.S., Identifikasi Iris Mata dengan Tapis Gabor Wavelet dan Jaringan Syaraf Tiruan Learning Vector Quantization (LVQ), Department of Electrical Engineering, Diponegoro University, Semarang, 2005.

[2] Gulmire, K. and S. Ganorkar, 2012. Iris Recognition using Gabor Wavelet, International Journal of Engineering Research and Technology (IJERT), vol 1, Issue 5: 1-5.

[3] Kusuma, A.A., 2009. Pengenalan Iris Mata Menggunakan Pencirian Matriks Ko-Okurensi Aras Keabuan, Department of Electrical Engineering, Diponegoro University, Semarang.

[4] Moreno R.P. and A. Gonzaga, 2009. Features Vector For Personal Identification Based On Iris Texture. Departamento de Engenharia Elétrica - EESC – USP, Februari.

[5] Daugman, J., 1993. "High Confidence Visual Recognition of Persons by a Test of Statistical Independence", IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 15, No.11: 1148-1161.

[6] Rahmawati, I., 2007. Pemampatan Citra Digital Dengan Wavelet Paket, Department of Electrical Engineering, Diponegoro University, Semarang.

[7] Center for Biometrics and Security Research, 2011. CASIA Iris Image Database, CASIA-IrisV1, Institute of Automation, Chinese Academy of Sciences, Beijing.

[8] ---, Image Processing Toolbox, for User's with MATLAB, User's Guide Version 3, The Mathwork Inc, 2001.

[9] Misiti, M., Y. Misiti, G. Oppenheim, and J.M. Poggi, 2004. Wavelet Toolbox For Use with Matlab, User's Guide Version 3, The Math Works, Natick, MA.

[10] Peng, T., 2007. Circular Hough Based on Gradient, Departement of Mechanical Engineering University of Maryland, College Park, Maryland, USA.

[11] Guido, R.C., JFW. Slaets, R. Koberle, LOB. Almeida, and J.C. Pereira, 2005. "A New Technique to Construct a Wavelet Transform Matching a Specified Signal with Applications to Digital, Real Time, Spike, and Overlap Pattern Recognition", Journal of Digital Signal Processing, vol 16, issue 1: 24-44.

[12] Gupta, A., S.D. Joshi, S. Prasad, 2002, "On a New Approach for Estimating Wavelet Matched to Signal", Proceeding 8th National Conference on Communications, IIT, Bombay: 180-184.

[13] Chui, C.K., 1997. Wavelets: A Mathematical Tool for Signal Analysis, Society for Industrial and Applied Mathematics, Philadelphia.

[14] MathWorks, 2012. Optimization Toolbox™ User's Guide, online only, Revised for Version 6.2.1 (Release 2012b), The Math Works, Natick, MA.