Analysis of Reinforced Concrete Capacity for Irregular Cross-Sections Using Numerical Methods

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Abstract Although reinforced concrete member usually forms the rectangular shape, for architectural reasons or an optimization purpose the section may be formed on the nonrectangular or irregular shape. Analysis of the sectional capacity in the case of non-square or irregular sections is very complex and takes time. This paper offers a cross-sectional analysis with a numerical approach that is implemented in a computer program. Several cross-sectional forms from the results of previous studies were adopted for validation purposes. The selection of the cross-section considers the representation of the cross-section shape and the configuration of the reinforcement. In addition to the cross-sectional shape, the method of analysis is also a consideration for selection. From the results of the analysis using numerical methods and comparison of the analysis of previous researchers, it shows a fairly good level of accuracy with an average deviation of 2.35%. The largest deviation was in Section 7 and 8 with a deviation of 5.03 and 8.62%, respectively. This deviation is more due to the analysis method. Analysis using finite element method for sections, 9 and 10 show slightly higher than the numerical method due to neglecting tensile strength of concrete and strain hardening of steel on the numerical method. On the other side, the analysis with the cross-section conversion approach shows lower results. This software may be used to solve the nonrectangular or irregular sections.

Keywords Irregular section · Numerical · Flexural

1 Introduction

In contrast to steel materials that have the same strength between compressive and tensile conditions. Generally, the tensile strength of concrete does not exceed 10% of the compressive strength [1, 2]. The low tensile strength of the concrete makes this material often combined with reinforcing steel to withstand the tension so that it forms a reinforced concrete composite structure. In general, the structural elements

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S. A. Kristiawan et al. (eds.), *Proceedings of the 5th International Conference on Rehabilitation and Maintenance in Civil Engineering*, Lecture Notes in Civil Engineering 225,
https://doi.org/10.1007/978-981-16-9348-9_6

of reinforced concrete are in the form of a square section. However, due to architectural rea-sons or structural optimization considerations, reinforced concrete sections can be non-square, such as circle, octagonal, hexagonal, trapezoidal, and triangular. Moreover, a square section that is subjected to the biaxial moment can also behave as a non-square section due to the rotation of the section in receiving the biaxial resultant moment.

In a bending element, the internal forces acting in a section will take balance, so that the compressive forces will be the same as the tensile forces. The compressive force in the cross-section is generated from compressed concrete and compressive reinforcement, while the tensile force is generated from tensile reinforcement. The compressive force of the concrete is the resultant stress of the concrete which forms a stress block. By assuming the plane section remains plane before and after bending, the strain distribution from the neutral axis to the extreme compression fiber can be considered a linear function of the neutral axis distance. Therefore, the stress distribution will also be identical to the shape of the stress–strain relationship curve. A stress block in a reinforced concrete section is a volume formed by the compression area of the concrete and its stress. In a square section where the width of the compression area is constant, the concrete compressive force of the stress block can be simplified to be an equivalent stress block in a square shape [3]. However, for non-square sections, the simplification of square stress blocks to calculate the compressive force of concrete may be wrong.

The calculation of non-square sections will be more appropriate if using the numerical approach than the stress block simplification. This paper discusses the moment capacity analysis on irregular sections of reinforced concrete beam by using a numerical approach implemented in a computer program to overcome design problems, especially in the case of non-square beam sections or biaxial bending cases.

1.1 Material Model

1.1.1 Concrete Compressive Stress–Strain

The compressive strength of concrete is generally determined from the compressive test of a concrete cylinder with 150 mm diameter and 300 mm length in the longitudinal direction at the age of 28 days. Numerous approaches are used to determine the shape of the concrete compressive stress–strain relationship curve before maximum stress in a second-order parabolic [4]. Hognestad proposed the compressive stress–strain curve of concrete as a parabolic function up to the maximum stress and subsequently decreasing linearly until it reaches the ultimate strain as shown in Fig. 1a [5]. The concrete compressive Stress–strain curve is described in two regions as shown in Eqs. 1 and 2.

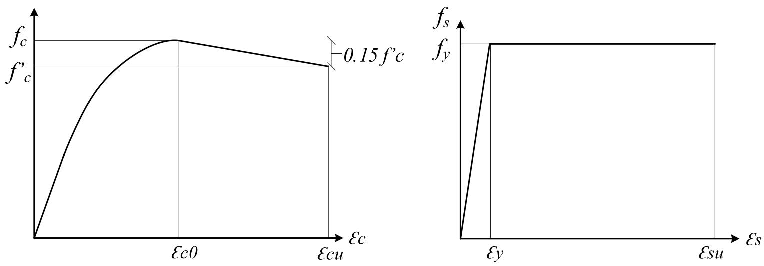

Fig. 1 Stress–strain relationship. **a** Concrete in compression. **b** Steel

$$f_c = f'_c\left[\frac{2\varepsilon_c}{\varepsilon_0} - \left(\frac{\varepsilon_c}{\varepsilon_0}\right)^2\right], 0 \geq \varepsilon_c \geq \varepsilon_{c0} \tag{1}$$

$$f_c = \left(\frac{0.15\varepsilon_c + 0.85\varepsilon_{co} - \varepsilon_{cu}}{\varepsilon_{co} - \varepsilon_{cu}}\right) f'_c, \varepsilon_{c0} \geq \varepsilon_c \geq \varepsilon_{cu} \tag{2}$$

Where, $\varepsilon_0 = \frac{2f'_c}{E_c}$ and $\varepsilon_{cu} = 0.0038$.

1.1.2 Steel

In general, the behavior of the steel is determined from the stress–strain relationship of the uniaxial tensile test results in the laboratory. The stress–strain relationship curves of steel for the compressive conditions are considered to be the same and identical to the tensile [4]. Chen determined the stress–strain relationship of steel by dividing three parts, the linear elastic region starting from the point of origin until the steel reaches the yield strain ε_y, horizontal plateau starting from yield-strain ε_y up to 8–15 times its elastic range ε_y, and strain hardening where the steel stress increases and reaches a maximum at the ultimate strain then gradually decreases until the failure strain [6]. In this study, the behavior of the stress–strain relationship of reinforcing steel is modeled as a bilinear function with neglecting the strain hardening effect as shown in Fig. 1b.

1.1.3 Section Model and Reinforcement Configuration

The irregular sections are modeled by multilinear from nodal points coordinate forms closed polygon. The coordinates of each polygon boundary point are denoted ($\mathbf{X_{(i)}}$, $\mathbf{Y_{(i)}}$), where **i** = 1, 2, 3, … n, and **n** is the number of polygon coordinates. The configuration and location of the reinforcement in the section are also determined by coordinates ($\mathbf{X_{r(i)}}$, $\mathbf{Y_{r(i)}}$), where **i** = 1, 2, 3, … nr, and **nr** is the number of reinforcements in the section. From the cross-sectional coordinates, it can be determined Y_{max} which is the outermost compressive point that undergoes the maximum compressive

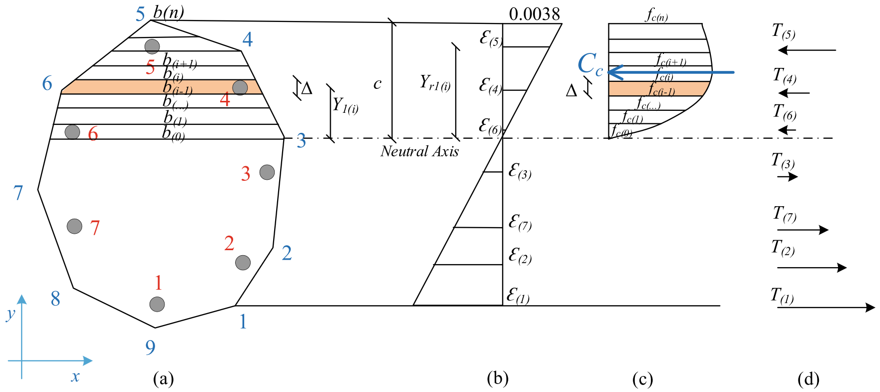

Fig. 2 **a** Section. **b** Strain diagram. **c** Concrete stress block. **d** Forces of reinforcements

strain. Meanwhile, from the coordinates of the location of the reinforcement can be determined the outermost tensile reinforcement. Irregular cross-sectional shape and reinforcement configuration as shown in Fig. 2a.

2 Cross Section Analysis

In the analysis of flexural beam with a rectangular section, the compressive force of concrete, and the tensile force of the reinforcement can be obtained from the equilibrium of forces, where the compressive force $\boldsymbol{C_c}$ is defined based on a simplified equivalent stress block of concrete. While the irregular cross-section, determination of compressive force $\boldsymbol{C_c}$ becomes very complex because it has to be solved through the integration of the stress–strain relationship function on the compression concrete area. Therefore, it will be more accurate in case irregular cross-section analysis undertakes numerically. The analysis of reinforced concrete sections in this study is based on several basic assumptions on the following flexural sections [7].

1. The first presumption is plane sections remain plane before and after bending as Bernoulli's principle. This assumption implies that the strain in the section is proportional to the distance to the neutral axis. The result of this assumption is that the distribution of compressive stress in the compressive region is identical to the stress-strain relationship curve.
2. The bond between concrete and steel reinforcement is assumed as full-bonded, and there is no slip between both concrete and steel. It means that the strain of reinforcement is the same as concrete strain at the same level.
3. The tensile stress of the concrete is not more than10% of its compressive strength, thus the force of the tensile concrete below the neutral axis is also small. The lever arm of the concrete tension force to the neutral axis is small. So that, the contribution of concrete tension force to the bending capacity of

the cross-section is considered very small. For this reason, the tensile stress of concrete can be neglected.

4. Concrete is considered only to be able to withstand compressive stress until the ultimate strain. ACI sec. 10.2.3 defined the ultimate compressive strain of the concrete to 0.003 for design [8]. However, in this study, the ultimate strain of compression concrete was determined according to Hognestad's model, i.e., 0.0038.

Resultant of internal forces in a section can be derived by Eq. 3.

$$P_t = C_c + \sum_{i=1}^{nr} T_{(i)} \tag{3}$$

Where:

C_c: Concrete compression force.

$\sum_{i=1}^{nr} T_{(i)}$: Sum of the reinforcement forces, where $\boldsymbol{nr}$ is number of reinforcement.

The resultant of internal forces $\boldsymbol{P_t}$ in Eq. 3 are obtained by determining the value of the outer tensile reinforcement as shown in Fig. 3. The process of determining $\boldsymbol{P_t}$ is carried out with the following procedure.

1. Define the strain of the outer tensile reinforcement $\boldsymbol{\varepsilon_s}$.
2. Assuming the strain distribution is linear, the depth of the neutral axis $\boldsymbol{c}$ can be defined. (see Fig. 2b).
3. The compressed concrete area is divided into small slices extending in the **X** direction in a number of $\boldsymbol{ns}$ slices. The thickness of each slice is $\boldsymbol{\Delta = c/ns}$.
4. Calculate the coordinates of the intersection points between the edge of slice and the polygon to calculate the width of the slice. $b_{(i)}$, $\boldsymbol{i} = 0, 1, 2, \ldots, \boldsymbol{ns}$, and $\boldsymbol{ns}$ is number of slices.

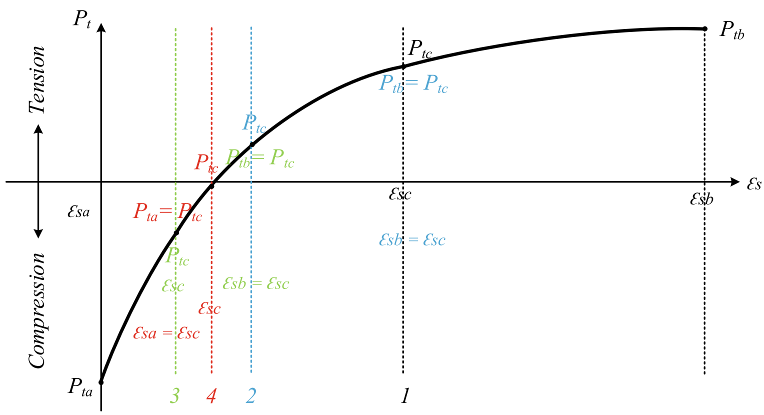

Fig. 3 Bisection iteration

5. Average slice width can be derived using Eq. 4.

$$b_{1(i)} = \frac{b_{(i-1)} + b_{(i)}}{2} \tag{4}$$

6. The center of gravity of each slice can be assumed exist at midpoint of slice thickness, thus the arm of these points to neutral axis is approximated by Eq. 5.

$$Y_{(i)} = (i - 0.5)\Delta \tag{5}$$

7. The concrete compressive strain on each slice edge is proportional to the distance of the edge to neutral axis, and the strain can be defined by Eq. 6.

$$\varepsilon_{c(i)} = \frac{i}{n} \times 0.0038 \tag{6}$$

   Where: $\boldsymbol{i} = 0, 1, 2, \ldots, \boldsymbol{ns}$, $\boldsymbol{ns}$ is number of slices
8. Furthermore, the compressive stress of the concrete on each slice edge can be determined based on the stress-strain relationship according to Eqs. 1 and 2.
9. The average stress of slice can be calculated with Eq. 7.

$$f_{c1(i)} = \frac{f_{c(i-1)} + f_{c(i)}}{2} \tag{7}$$

10. Compression force for each slice can be defined by Eq. 8.

$$C_{c1(i)} = \left(b_{1(i)} \times \Delta\right) f_{c1(i)} \tag{8}$$

11. The resultant of concrete compressive force $\boldsymbol{C_c}$ is the sum of all the compressive forces of the on each slice as shown in Eq. 9.

$$C_c = \sum_{i=1}^{n} C_{c1(i)} \tag{9}$$

12. The space of the reinforcement to the neutral axis is determined based on Eq. 10. The negative sign indicates the location of the reinforcement is below the neutral axis.

$$Y_{r1(i)} = Y_{r(i)} + c - Y_{max} \tag{10}$$

13. The reinforcement strain proportional with respect to the distance of the reinforcement to the neutral axis, the reinforcing strain can be calculated by Eq. 11. In this case the negative sign indicates the tensile strain.

$$\varepsilon_{s(i)} = \frac{0.0038}{c} Y_{r1(i)} \tag{11}$$

14. Base on Eq. 11, then reinforcement stresses in each reinforcing bar is obtained using expression of Eq. 12.

$$f_{s(i)} = \varepsilon_{s(i)} \times E_s, \text{ where } E_s = 2 \times 10^5 \text{Mpa} \tag{12}$$

if $\varepsilon_{s(i)} > \varepsilon_y$, then $f_{s(i)} = f_y$.
if $\varepsilon_{s(i)} < -\varepsilon_y$, then $f_{s(i)} = -f_y$.

15. The forces on each reinforcement are the product of the reinforcement area and the stress as shown in Eq. 13.

$$T_{(i)} = A_{s(i)} \times f_{s(i)} \tag{13}$$

16. From the compression force C_c and the reinforcement forces $T_{(i)}$ can be calculated the resultant internal forces of section using Eq. 3.

2.1 Internal Forces Equilibrium

To achieve a balance of internal forces of the cross section can be obtained iteratively by using the bisection method. The concrete strain at top fiber is set 0.0038, and the outer reinforcing steel strain is entered twice, i.e., $\boldsymbol{\varepsilon_{sa}} = 0$ and $\boldsymbol{\varepsilon_{sb}} = 0.04$ as the initial strain. Iteration process of bisection method as shown in Fig. 3.

1. The first step is to enter $\boldsymbol{\varepsilon_{sa}} = 0$. By using the calculation procedure to find the resultant of internal forces for $\boldsymbol{\varepsilon_s} = \boldsymbol{\varepsilon_{sa}}$ will be found $\boldsymbol{P_{ta}}$ that produce the compressive force, (see Fig. 4c).

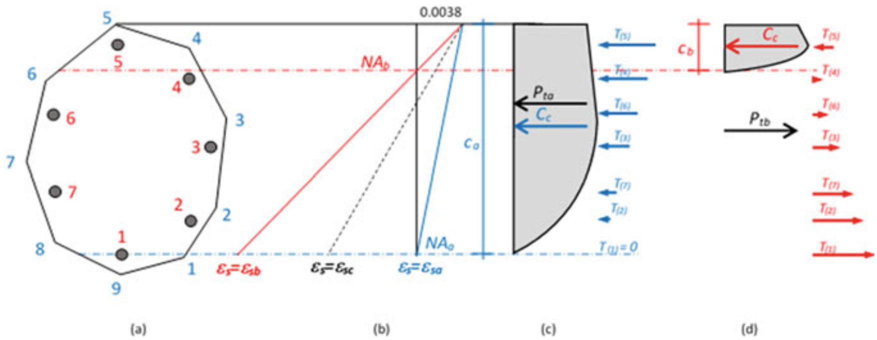

Fig. 4 **a** Section. **b** Strain iteration. **c** Internal forces at $\varepsilon_s = \varepsilon_{sa}$. **d** Internal forces at $\varepsilon_s = \varepsilon_{sb}$

2. By using the same procedure enter $\boldsymbol{\varepsilon_{sb}} = 0.04$ as $\boldsymbol{\varepsilon_s}$ to get $\boldsymbol{P_{tb}}$ that produce the force in tension, (see Fig. 4d). Where $\boldsymbol{\varepsilon_{sa}}$ and $\boldsymbol{\varepsilon_{sb}}$ are the initial strain given to outer reinforcement which has not produce the balance condition ($\boldsymbol{P_t} = 0$).
3. The Next Step Using $\boldsymbol{\varepsilon_{sc}}$ as Mid-Point Between $\boldsymbol{\varepsilon_{sa}}$ and $\boldsymbol{\varepsilon_{sb}}$, $\varepsilon_{sc} = \left(\frac{\varepsilon_{sa}+\varepsilon_{sb}}{2}\right)$. Then Calculate the Resultant of Internal Forces for $\boldsymbol{\varepsilon_s = \varepsilon_{sc}}$, i.e., $\boldsymbol{P_{tc}}$.
   If $P_{tc} \times P_{ta} < 0$, then $\boldsymbol{\varepsilon_{sb} = \varepsilon_{sc}}$.
   If $P_{tc} \times P_{tb} < 0$, then $\boldsymbol{\varepsilon_{sa} = \varepsilon_{sc}}$.
4. If $\left|\frac{(\varepsilon_c - \varepsilon_a)}{\varepsilon_c}\right| \leq Tolerance$ or $\left|\frac{(\varepsilon_c - \varepsilon_b)}{\varepsilon_c}\right| \leq Tolerance$, then go to step 5. If No, back to step 3.
5. The $\boldsymbol{\varepsilon_s = \varepsilon_{sc}}$, in this iteration the equilibrium occurs.

The balanced condition occurs when the strain of outer tension reinforcement ε_s produces internal forces resultant P_t which very small or less than the tolerance value defined earlier. Then calculate the concrete compressive moment from the number of slice forces multiplied by its lever arm to the neutral axis as shown in Eq. 14, and the moment of reinforcement which is the result of an addition of moment of each reinforcement to the neutral axis as shown in Eq. 15.

$$M_c = \sum_{i=1}^{ns} C_{c1(i)} \times Y_{1(i)} \tag{14}$$

$$M_r = \sum_{i=1}^{nr} T_{(i)} \times Y_{r1(i)} \tag{15}$$

The flexural capacity can be determined by using Eq. 16

$$M = M_c + M_r \tag{16}$$

Numerical solution to determinate the reinforced concrete capacity presented in Fig. 5.

3 Program Validation

Before the program is used, a validation process is required to ensure whether the program is correct or still requires improvement. In this study, the validation process was undertaken by comparing the results of program running and analysis from previous studies. The sections used for validation represent the geometric shape of the square and irregular sections. Reinforcement ratio and reinforcement configuration are also considered in section selection. The selected sections in this study as shown in Fig. 6 (Table 1).

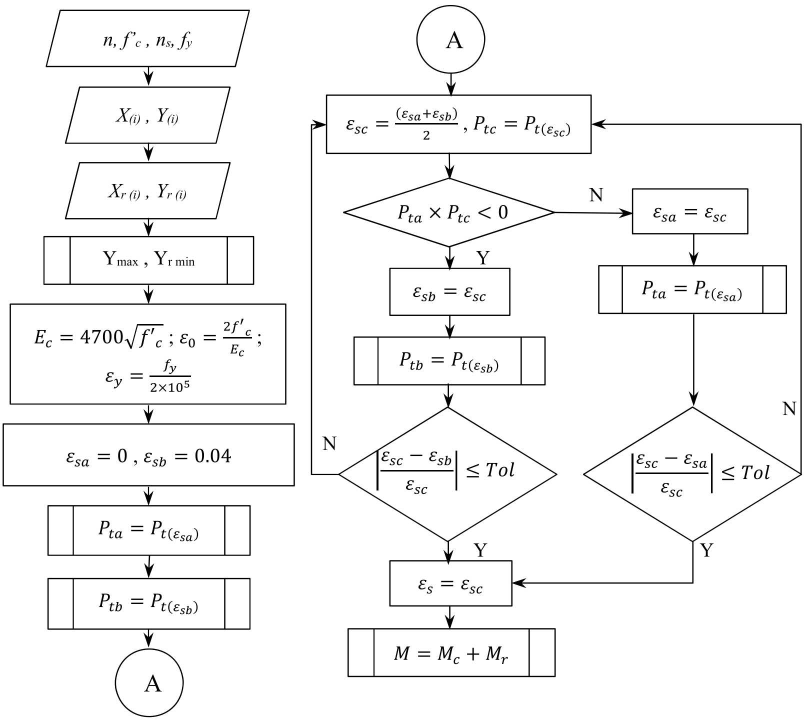

Fig. 5 Flow chart of irregular reinforced concrete beam section analysis

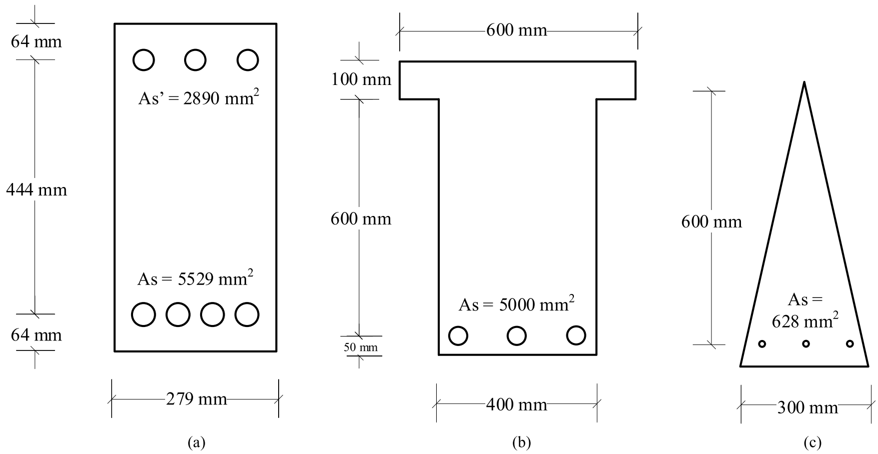

Fig. 6 Selected cross section of beam

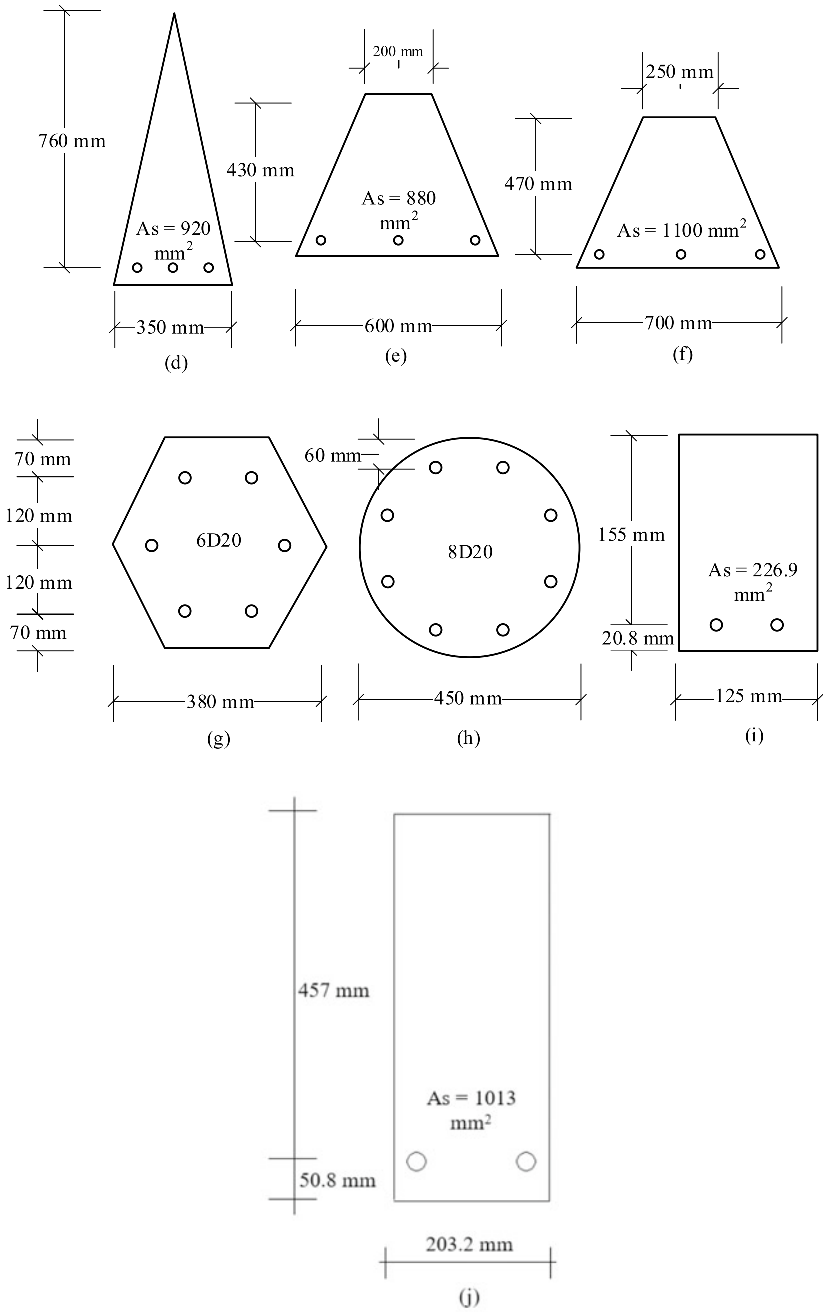

Fig. 6 (continued)

Table 1 The results of analysis

No	Section	f'_c (MPa)	f_y (MPa)	ϕ μ (kN.m)		Deviation (%)	Analysis method
				Analysis	Reference		
1	Figure 6a	20.7	276	603.31	595.00	1.40	Approximation analysis [4]
2	Figure 6b	30	400	1141.14	1139.00	0.19	Flexural equation [9]
3	Figure 6c	30	420	108.56	109.00	0.41	Flexural equation [9]
4	Figure 6d	30	420	202.22	203.00	0.38	Flexural equation [9]
5	Figure 6e	30	420	131.99	132.00	0.00	Flexural equation [9]
6	Figure 6f	30	420	181.45	180.00	0.81	Flexural equation [9]
7	Figure 6g	30	400	93.38	88.90	5.03	Equivalent square shape [9]
8	Figure 6h	30	400	152.72	140.60	8.62	Equivalent square shape [9]
9	Figure 6i	41.8	400	11.88	12.10	1.84	Finite element analysis [10]
10	Figure 6j	33.23	309.5	122.36	128.50	4.78	Finite element analysis [10]

4 Result and Discussion

Based on the numerical analysis results and comparing with the previous studies, it is apparent that the cross-sectional analysis using the numerical method implemented in a computer program is very accurate with an average deviation of 2.35%.

The largest deviation in this study is the comparison between cross-sections 7 and 8 analysis in this study and reference with deviation 5.03 and 8.62%, respectively. These deviations are more due to the analysis method distinction, where hexagonal and circular sections are simplified with square sections. Cross-sections analyzed with FEM i.e., 9 and 10 indicate greater capacity than this study. This is because the numerical analysis of cross-section ignores the concrete tensile stress as stated in one of the analysis assumptions. The cross-sections 7 and 8 analyzed with the section area approximation, where the hexagonal and circular sections are approached by converting the square section to simplify the analysis. The other cross-sections analyzed with flexural section analysis show a very small deviation i.e., 0.36%.

A similar numerical simulation was also developed by Dundar and Sahin that assume the irregular cross-sections as a closed polygon [11]. Even though the simulation model can analyze arbitrary reinforced concrete sections subjected to axial load and biaxial bending. But, the model still uses an equivalent stress block to compute concrete compression force that may be inappropriate for the non-square section. On the other hand, Oscar Fitrah Nur undertook a numerical analysis of rectangular reinforced concrete beam with divide compression area into small slices to generate the stress block which appropriates with actual stress–strain [12].

The simulation in this study considers the cross-section as a closed polygon and the stresses in compression concrete as a function of strain which linear to the distance of the neutral axis. So, this study indicates that the numerical simulations implemented in a computer program show progress in research.

5 Conclusion

The numerical method by using a multi-slice approach with bisection iteration to calculate the moment capacity of the reinforced concrete section shows a good result with an average deviation of 2.35% compared with other methods. Implementation of this method as a computer program is very precise to analyze reinforced concrete beams for the irregular section including circular, hexagonal, T shapes, etc.

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