Construction of Hyperplane, Supporting Hyperplane, and Separating Hyperplane on Rn and Its Application

by Susilo Hariyanto

Construction of Hyperplane, Supporting Hyperplane, and Separating Hyperplane on $\mathbb{R}^n$ and Its Application

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Abstract. Hyperplane, supporting hyperplane and separating hyperplane have been defined well in inner product space. These definitions are expressed in very general concept of space, so in its understanding it requires understanding the specific inner product space. It is difficult, so in this paper the definitions will be explained in the $\mathbb{R}^n$ space or Euclidean space, beginning with constructing all possible hyperplanes of a given convex set. From all the hyperplane constructions, they will be classified as supporting hyperplane or separating hyperplane. This understanding will be generalized in the case of convex set with point and with other convex set. To clarify this discussion, it is completed with several examples of the construction of the hyperplane in $\mathbb{R}^n$. In addition, this paper will give some examples of the application of hyperplane construction in $\mathbb{R}^n$ in computing the distance between point and a convex set. The application will also be further discussed in a more general problem, namely the distance between two disjoint convex sets.

1. Introduction

Let $\mathcal{Y}$ be a set in a linear space and $\boldsymbol{z}$ be a point outside $\mathcal{Y}$. Since $\boldsymbol{z} \notin \mathcal{Y}$, it is possible to construct a hyperplane that separate $\mathcal{Y}$ and $\boldsymbol{z}$. In this paper, it is shown the definition about hyperplane, supporting hyperplane and separating hyperplane, which usual defined on inner product space. Hyperplane is a set whose membership requirements are determined by a particular vector and scalar. This abstract definition has been clearly stated in [1], [2] and [3]. In addition, these literatures also explain the notion of a supporting and separating hyperplane. Both of them are still quite difficult to visualize in our understanding. This is interesting to be used as research material that aims to simplify in understanding of the types of hyperplane.

This research has been studied by [4] who examines the problem of constructing a family of hyperplanes that separates two disjoint polyhedral. In several articles, separating hyperplane is constructed to determine the distance of two objects, such as [5], that constructed a separating hyperplane to determine the distance between two ellipsoids, and [6] who made separating and supporting hyperplane to compute the distance between two convex sets, by using minimum duality theorem. Based on these researches, in this paper it is shown the hyperplane construction and classify them in to supporting and separating hyperplane, but it is restricted to convex sets in $\mathbb{R}^n$.

The plan of this paper is as follows. In section 1 contains necessary background. Section 2 explains the definition of hyperplane, separating hyperplane and supporting hyperplane. In section 3, we will show an example of separating hyperplane and supporting hyperplane and in section 4 we give the

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application to compute the distance between two sets. We choose the convex sets to guarantee the distance of the sets, [7].

2. Hyperplane

In this section, we give some assertions of hyperplane, separating hyperplane and supporting hyperplane in abstract terms. In this section and so on, we consider $\mathcal{Y}$ be a nonempty convex set in Hilbert space $\mathbb{H}$. Definition 1 shows the definition of hyperplane.

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Definition 1. [3, 2] *In an inner product space* $\mathbb{H}$, *A hyperplane in* $\mathbb{H}$ *is a set whose membership requirements are determined by vector* $\boldsymbol{a}$ *scalar* α *that satisfy* $\langle \boldsymbol{a}, \boldsymbol{x} \rangle = \alpha$. *Then this hyperplane is denoted by* $\mathcal{H}_{a,\alpha} = \{\boldsymbol{x} | \langle \boldsymbol{a}, \boldsymbol{x} \rangle = \alpha\}$.

Any hyperplane $\mathcal{H}_{a,\alpha}$ divides linear space into the two closed halfspaces

$$\mathcal{H}^{\leq}_{\boldsymbol{a},\alpha} = \{\boldsymbol{x} | \langle \boldsymbol{a}, \boldsymbol{x} \rangle \leq \alpha\}$$
$$\mathcal{H}^{\geq}_{a,\alpha} = \{\boldsymbol{x} | \langle \boldsymbol{a}, \boldsymbol{x} \rangle \geq \alpha\}.$$

If the boundary line $\langle \boldsymbol{a}, \boldsymbol{x} \rangle = \alpha$ is excluded, then we have the two open halfspaces

$$\mathcal{H}^{<}_{\boldsymbol{a},\alpha} = \{x | \langle \boldsymbol{a}, \boldsymbol{x} \rangle < \alpha\}$$
$$\mathcal{H}^{>}_{\boldsymbol{a},\alpha} = \{x | \langle \boldsymbol{a}, \boldsymbol{x} \rangle > \alpha\}$$

For simplicity, it is written $\mathcal{H}$ instead of $\mathcal{H}_{\boldsymbol{a},\alpha}$. From Definition 1, we derive this below theorem that state Next, the definition of supporting hyperplane and separating hyperplane are given below.

Theorem 2. (Separating Hyperplane) [1] *Suppose* $\mathcal{Y}$ *be a convex set and* $\boldsymbol{z} \in \mathbb{H}$ *be a point outside* $\mathcal{Y}$ *then there exist* $\boldsymbol{a} \neq \boldsymbol{0}$ *and scalar* α *such that* $\langle \boldsymbol{a}, \boldsymbol{x} \rangle \leq \alpha, \forall \boldsymbol{x} \in \mathcal{Y}$ *and* $\langle \boldsymbol{a}, \boldsymbol{z} \rangle \geq \alpha$.

Theorem 3 below guarantees the uniqueness of projection to a convex set.

Theorem 3. (The Projection Theorem) [8, 6] *Let* $\boldsymbol{z} \in \mathbb{H}$ *be some point outside* $\mathcal{Y}$, *there exists a unique point* $\boldsymbol{x}^* \in \mathcal{Y}$ *that satisfies the following properties*

$$\|\boldsymbol{z} - \boldsymbol{x}^*\| = \inf_{\boldsymbol{x} \in \mathcal{Y}} \|\boldsymbol{z} - \boldsymbol{x}\|$$

and

$$\langle \boldsymbol{z} - \boldsymbol{x}^*, \boldsymbol{x} - \boldsymbol{x}^* \rangle \leq 0, \forall x \in \mathcal{Y}$$

The point $\boldsymbol{x}^*$ is called the projection of $\boldsymbol{z}$ to $\mathcal{Y}$. Definition 4 state the definition of supporting hyperplane.

Definition 4. (Supporting Hyperplane) [1] *Suppose* $\mathcal{Y}$ *is a convex set in* $\mathbb{R}^n$, *and* $\boldsymbol{x}^* \in \mathcal{Y}$. *Then the hyperplane* $\mathcal{H} = \{\boldsymbol{x} | \langle \boldsymbol{a}, \boldsymbol{x} \rangle = \langle \boldsymbol{a}, \boldsymbol{x}^* \rangle\}$ *is supporting* $\mathcal{Y}$ *on* $\boldsymbol{x}^*$.

It is equivalent to say that the point $\boldsymbol{x}^*$ and the set $\mathcal{Y}$ are separated by the hyperplane $\mathcal{H}$. The geometric interpretation is that the hyperplane $\mathcal{H}$ is tangent to $\mathcal{Y}$ at $\boldsymbol{x}^*$ and halfspace $\{\boldsymbol{x} | \langle \boldsymbol{a}, \boldsymbol{x} \rangle \leq \langle \boldsymbol{a}, \boldsymbol{x}^* \rangle\}$ contains $\mathcal{Y}$.

Ref. [6] state that hyperplane $\mathcal{H}$ is supporting hyperplane of $\mathcal{Y}$ if

$$\sup_{\boldsymbol{x} \in \mathcal{Y}} \langle \boldsymbol{a}, \boldsymbol{x} \rangle = \alpha$$

Since $\boldsymbol{x}^*$ is contained on hyperplane $\mathcal{H}$, and there exits $\boldsymbol{x} \in \mathcal{Y}$, then hyperplane $\mathcal{H}$ is supporting $\mathcal{Y}$ if $\boldsymbol{x}^* = \boldsymbol{x}$. The function

$$\alpha(\boldsymbol{a}) = \sup_{\boldsymbol{x} \in \mathcal{Y}} \langle \boldsymbol{a}, \boldsymbol{x} \rangle$$

is called the support function of $\mathcal{Y}$. Next, we derive this corollary from theorem 2 and definition 4, where $\boldsymbol{z}$ is a boundary point of $\mathcal{Y}$.

Corollary 5. [6] (Existence of Supporting Hyperplane) *Let* $\boldsymbol{z}$ *be a boundary point of* $\mathcal{Y}$. *Then there exists a vector* $\boldsymbol{a} \in \mathbb{H}$ *such that*

$$\sup_{\boldsymbol{x} \in \mathcal{Y}} \langle \boldsymbol{a}, \boldsymbol{x} \rangle = \langle \boldsymbol{a}, \boldsymbol{z} \rangle$$

Then the hyperplane $\mathcal{H} = \{x | \langle \boldsymbol{a}, \boldsymbol{x} \rangle = \langle \boldsymbol{a}, \boldsymbol{z} \rangle\}$ *supports* $\mathcal{Y}$ *at the point* z.

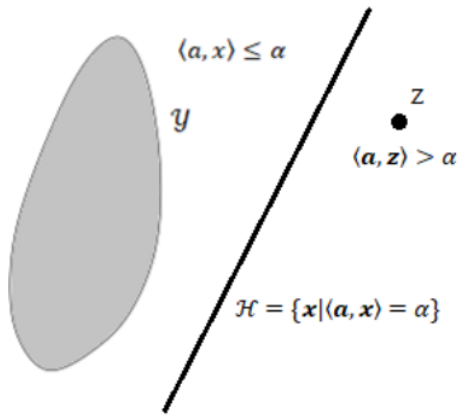

Figure 1 Hyperplane $\boldsymbol{\mathcal{H}} = \{\boldsymbol{x} | \langle \boldsymbol{a}, \boldsymbol{x} \rangle = \boldsymbol{\alpha}\}$ separates convex set $\boldsymbol{\mathcal{Y}}$ and point $\boldsymbol{z}$.

In Figure 1, it is shown that convex sets $\mathcal{Y}$ and vector $\boldsymbol{z}$ are separated by hyperplane $\mathcal{H}$. Next section, it is shown the construction of separating and supporting hyperplane.

3. Distinguishing The Separating and Supporting Hyperplane

Section 1 shows that any hyperplane $\mathcal{H}$ divides linear space into two halfspaces. Since we consider this in $\mathbb{R}^n$ or Euclidean space, then $\langle \boldsymbol{x}, \boldsymbol{y} \rangle = \boldsymbol{x}^T \boldsymbol{y}$, for $\boldsymbol{x}$ and $\boldsymbol{y}$ are vectors in $\mathbb{R}^n$.

As example, in $\mathbb{R}^2$ it is given a circle $\mathcal{Y} \equiv x^2 + y^2 = 1$ and a point $z = (3,3)$. Some hyperplanes that can be constructed are:

a. $\mathcal{H}_1 \equiv x + y = \sqrt{2}$,
b. $\mathcal{H}_2 \equiv x + y = 4$,
c. $\mathcal{H}_1 \equiv x + y = 8$.

According to [6], hyperplane $\mathcal{H} = \{\begin{pmatrix} x \\ y \end{pmatrix} | a^T \begin{pmatrix} x \\ y \end{pmatrix} = \alpha\}$ is called to be supporting hyperplane of convex set $\mathcal{Y}$ if

$$\sup_{(\mathrm{x},\mathrm{y}) \in \mathcal{Y}} a^T \begin{pmatrix} x \\ y \end{pmatrix} = \alpha$$

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To classify these hyperplanes, we choose the boundary point of $\mathcal{Y}$. If we transform the Cartesian to polar coordinates, we obtained the circle equation is $\mathcal{Y} \equiv r = 1$, and the outer point of $\mathcal{Y}$ is $(\cos\theta, \sin\theta)$.

a. For hyperplane $\mathcal{H}_1 \equiv x + y = \sqrt{2}$,

$$\cos\theta + \sin\theta = \sqrt{2}$$
$$\rightarrow 1 + \sin 2\theta = 2$$
$$\rightarrow \sin 2\theta = 1$$
$$\rightarrow \theta = \frac{\pi}{4}$$

Then, it is found the point $(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$ that contained on circle $\mathcal{Y}$ and also in hyperplane $\mathcal{H}_1$. So, hyperplane $\mathcal{H}_1 \equiv x + y = \sqrt{2}$ is supporting hyperplane of circle $\mathcal{Y}$, and point $z = (3,3)$ lies on its positive halfspace, because $3 + 3 \geq \sqrt{2}$.

b. For hyperplane $\mathcal{H}_2 \equiv x + y = 4$,

$$\cos\alpha + \sin\alpha = 4$$

It is impossible, since the maximum value of $\cos\alpha$ dan $\sin\alpha$ is 1, and the maximum value of $\cos\alpha + \sin\alpha$ is $\sqrt{2}$, then $\cos\alpha + \sin\alpha \leq 4$ and for $z = (3,3)$ we obtain $3 + 3 \geq 4$. Then $\mathcal{H}_2$ is separating hyperplane.

c. For hyperplane $\mathcal{H}_3 \equiv x + y = 8$,

$$\cos\alpha + \sin\alpha = 8$$

It is impossible, since the maximum value of $\cos\alpha$ dan $\sin\alpha$ is 1. And the maximum value of $\cos\alpha + \sin\alpha$ is $\sqrt{2}$, then $\cos\alpha + \sin\alpha \leq 4$ and for $z = (3,3)$ we obtain $3 + 3 \leq 8$. Then, hyperplane $\mathcal{H}_3$ is not separating hyperplane nor supporting hyperplane, because circle $\mathcal{Y}$ and **z** lie on the negative of halfspace $\mathcal{H}_3$.

The position of the three hyperplane is shown on Figure 2.

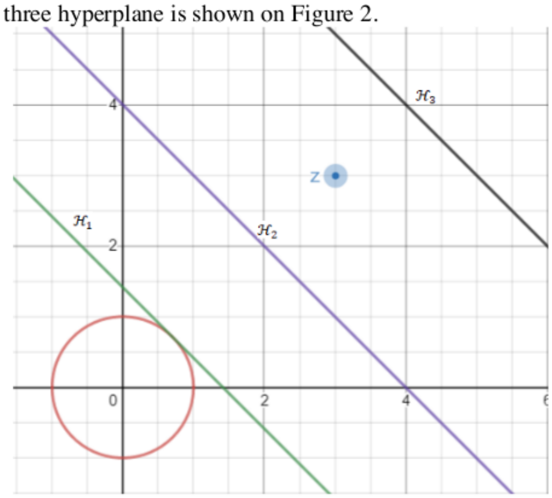

Figure 2 Supporting Hyperplane and Separating Hyperplane

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4. Application

In this section it is given the construction of hyperplane and its application for computing the distance between two convex set. The reader is referred to [6] for detailed discussion of this problem.

Let $\mathcal{H}_\alpha$ be a hyperplane in $\mathbb{R}^n$, that denoted as a set

$$\mathcal{H}_\alpha = \{\boldsymbol{x} | \langle \boldsymbol{a}, \boldsymbol{x} \rangle = \alpha\}$$

The distance between $\boldsymbol{z}$ and $\mathcal{H}$ is defined as

$$\text{dist}(\boldsymbol{z}, \mathcal{H}) = \inf_{\boldsymbol{x} \in \mathcal{H}_\alpha} \|\boldsymbol{z} - \boldsymbol{x}\|$$

where $\boldsymbol{a} \in \mathbb{R}^n$ and $\alpha \in \mathbb{R}$. Let $\boldsymbol{z} \in \mathcal{H}^{\geq}$, according to [9], the distance between $\boldsymbol{z}$ and $\boldsymbol{\mathcal{H}}_\alpha$ is

$$\text{dist}(\boldsymbol{z}, \mathcal{H}) = \frac{\langle \boldsymbol{a}, \boldsymbol{z} \rangle - \alpha}{\|\boldsymbol{a}\|}$$

For detailed proof see [9]. Let $\mathcal{H}_\alpha = \{\boldsymbol{x} | \langle \boldsymbol{a}, \boldsymbol{x} \rangle = \alpha\}$ and $\mathcal{H}_\beta = \{\boldsymbol{x} | \langle \boldsymbol{a}, \boldsymbol{x} \rangle = \beta\}$. The distance between these sets is defined as

$$\text{dist}(\mathcal{H}_\alpha, \mathcal{H}_\beta) = \inf\{\|\boldsymbol{x_1} - \boldsymbol{x_2}\| | \boldsymbol{x_1} \in \mathcal{H}_\alpha, \boldsymbol{x_2} \in \mathcal{H}_\beta\}$$

Therefore, the distance between $\mathcal{H}_\alpha$ and $\mathcal{H}_\beta$ is

$$\text{dist}(\mathcal{H}_1, \mathcal{H}_2) = \frac{\beta - \alpha}{\|\boldsymbol{a}\|}$$

In this subsection below, it is given some examples for computing the distance between two convex sets based on hyperplane construction.

a. The distance between ball and hyperplane.

Let $\mathcal{Y} = \{\boldsymbol{y_c} + r\boldsymbol{u} | \|\boldsymbol{u}\| \leq 1\}$ be a ball at center $\boldsymbol{y_c}$ and radius r, and $\mathcal{Z} = \{\boldsymbol{z} | \langle \boldsymbol{b}, \boldsymbol{z} \rangle = \alpha\}$ be a hyperplane. The first step to compute the distance between ball $\mathcal{Y}$ and hyperplane $\mathcal{Z}$ is determining the supporting hyperplane of $\mathcal{Y}$.

The supporting hyperplane that supports $\mathcal{Y}$ is given by

$$\sup_{\boldsymbol{y} \in \mathcal{Y}} \langle \boldsymbol{a}, \boldsymbol{y} \rangle = \langle \boldsymbol{a}, \boldsymbol{y_c} \rangle + r\|\boldsymbol{a}\|$$

Let $\boldsymbol{b} = k\boldsymbol{a}$, $k \in \mathbb{R}$, then

$$\mathcal{Z} = \{\boldsymbol{z} | \langle \boldsymbol{a}, \boldsymbol{z} \rangle = \frac{\alpha}{k}\}$$

Then the distance between ball $\mathcal{Y}$ and hyperplane $\mathcal{Z}$ is

$$\text{dist}(\mathcal{Y}, \mathcal{Z}) = \frac{\frac{\alpha}{k} - (\langle \boldsymbol{a}, \boldsymbol{y_c} \rangle + r\|\boldsymbol{a}\|)}{\|\boldsymbol{a}\|}$$

b. The distance between two ellipsoids.
Let $\mathcal{Y}$ and $\mathcal{Z}$ be two ellipsoids in $\mathbb{R}^n$ that denoted as

$$\mathcal{Y} = \{\boldsymbol{y_c} + B\boldsymbol{w} \mid \|\boldsymbol{w}\| \le 1\},$$
$$\mathcal{Z} = \{\boldsymbol{z_c} - C\boldsymbol{v} \mid \|\boldsymbol{v}\| \le 1\}$$

Where $\boldsymbol{y_c}$ and $\boldsymbol{z_c}$ are the centre, B, C are the matrices. The distance between $\mathcal{Y}$ and $\mathcal{Z}$ is attained by solving the minimum norm problem

$$\min \|\boldsymbol{y_c} - \boldsymbol{z_c} + B\boldsymbol{w} + C\boldsymbol{v}\|$$
$$\text{subject to} \|\boldsymbol{w}\| \le 1, \|\boldsymbol{v}\| \le 1$$

In this case, hyperplane construction is applied to dual problem of minimum distance. The supporting hyperplane of $\mathcal{Y}$ and $\mathcal{Z}$ is given by

$$\inf_{\boldsymbol{z} \in \mathcal{Z}} \langle \boldsymbol{a}, \boldsymbol{z} \rangle = \langle \boldsymbol{a}, \boldsymbol{z_c} \rangle - \langle C^{\dagger}\boldsymbol{a}, \boldsymbol{v} \rangle$$
$$= \langle \boldsymbol{a}, \boldsymbol{z_c} \rangle - \|C^{\dagger}\boldsymbol{a}\|$$

and

$$\sup_{\boldsymbol{y} \in \mathcal{Y}} \langle \boldsymbol{a}, \boldsymbol{y} \rangle = \langle \boldsymbol{a}, \boldsymbol{y_c} \rangle - \langle B^{\dagger}\boldsymbol{a}, \boldsymbol{w} \rangle$$
$$= \langle \boldsymbol{a}, \boldsymbol{y_c} \rangle - \|B^{\dagger}\boldsymbol{a}\|$$

Where $C^{\dagger}$ and $B^{\dagger}$ denote the adjoint of C and B respectively, if C and B are the operators. (In this case, since C and B are matrices, then $C^{\dagger}$ and $B^{\dagger}$ are the transpose respectively). So, the dual problem of the minimum distance of two ellipsoids is

$$\max \sigma(\boldsymbol{a}) = \langle \boldsymbol{a}, \boldsymbol{z_c} - \boldsymbol{y_c} \rangle - \|C^{\dagger}\boldsymbol{a}\| - \|B^{\dagger}\boldsymbol{a}\|$$
$$\text{subject to} \|\boldsymbol{a}\| \le 1$$

If $\mathcal{Y}$ turns to normed ball, then $B = \rho I$, where I is identity and ρ is radius. By putting Karush-Kuhn-Tucker condition, we can determine the distance between two normed balls is

$$\|\boldsymbol{y_c} - \boldsymbol{z_c}\| - (\rho_1 + \rho_2)$$

5. Conclusion

We have shown the construction of hyperplanes, separating hyperplanes and supporting hyperplanes. To compute the distance of point to convex sets, we must choose the supporting hyperplane that also separate the convex sets and points. In $\mathbb{R}^n$ space, the dimension of hyperplane is $n - 1$. If $n = 1$, the hyperplane is point, the hyperplane in $\mathbb{R}^2$ is line and in the $\mathbb{R}^3$ is plane. By visualizing it, it makes easier to understand.

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