□ – semigroup generated by a semigroup

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Γ– semigroup generated by a semigroup

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Abstract. In Γ– semigroup *S*, the element of Γ maybe a binary operation in *S*. Every element of any semigroup*S* can define a binary operation in *S*. A collection of binary operations defined of the element of S generates Γ– semigroup *S*.

1. Introduction

A semigroup is an algebraic structure consisting of a nonempty set equipped with an associative binary operation in the set. Semigroup plays an important role in some areas of mathematics, among others, in coding theory, combinatorics, and mathematical analysis. In 1986, Sen and Saha [1-3] defined an idea Γ– Semigroup which was a generalization of a semigroup. Some classical ideas from Semigroup have been developed in semigroups such as commutativity, regularity, and ideal [4-6] and many others.

In this article, Γ– semigroup *S* is viewed from the other side where Γ as a set of associative binary operations on *S*. This does not change the idea that has been developed on the semigroup defined by Sen. It means that all ideas on semigroups defined by Sen also apply to Γ– semigroup in this article.

It is interesting to be examined that each of Semigroup can generate Γ– semigroup. Any element of a semigroup *S* can define n associative binary operation on *S* such that a collection of associative binary operations on *S* Γ will be showed that S is a Γ–semigroup. Furthermore, we will review the properties f the Γ– Semigroup deprived of it's generating semigroup.

2. Γ–semigroup

Let *S* and Γ be two non-empty sets. S is called Γ –semigroup [1], [2], [3] if there exists a mapping $S \times \Gamma \times S \to S$ which written with $(x,\gamma,y) \to x\gamma y$ that satisfy $(x\gamma y)\beta z = x\gamma(y\beta z)$ for every $x,y,z \in S$ and for every $\gamma,\beta \in \Gamma$ .Some of the ideas of semigroup developed on Γ– semigroup *S* is said to be commutative if $x\gamma y = y\gamma x$ for every $x,y \in S$ and $\gamma \in \Gamma$. Element *a* in the Γ– semigroup *S* is said regular if there exists $x \in S$ such that $a = axa$ and *S* is called regular semigroup if for every element of *S* is regular. Element *a* in the Γ – semigroup is said regular if $a \in a\Gamma S\Gamma a$ and *S* is called regular Γ – semigroup if every element of S is regular. Element *a* in the semigroup S is said to complete regular if there exists $\alpha \in \Gamma$ and $b \in S$ such that $a = (a\alpha b)\alpha a$ and $a\alpha b = b\alpha a$. Γ – semigroup S is said to complete regular if every element of S is complete regular [4]. If *S* is a semigroup, m and n are nonnegative integers, $a \in S$ is said a (m,n) – regular if there exists $b \in S$ such that $a = a^m ba^n$. S is called (m,n) – regular semigroup if every element of *S*is a

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(m,n) – regular. (In this case, a^0 is defined with $a^0 b = b$ and $ba^0 = b$ .If S is a Γ – semigroup, m, n are non-negative integers, the element $a \in S$ is said (m,n) – regular if there exists $b \in S$ such that $a \in a^m \Gamma b \Gamma a^n$. Γ – semigroup S is called a (m,n) – regular Γ – semigroup if every element of S is (m,n) – regular.In this case, a^0 is defined with $a^0 \Gamma b = \{b\}$ and $b \Gamma a^0 = \{b\}$. [5]

$A, B \subseteq S$ is defined by $A\Gamma B = \{a\gamma b \mid a \in A, b \in B, \gamma \in \Gamma\}$ where Let S is Γ – semigroup. If $b \in S$ and $A\Gamma b = A\Gamma\{b\}, b\Gamma A = \{b\}\Gamma A$, the non-empty set $A \subseteq S$ is called Γ – subsemigroup of S if $A\Gamma A \subseteq A,$ a non-empty set $I \subseteq S$ is called left Γ – the ideal of S if for every $x \in I, \gamma \in \Gamma$ dan $b \in S, x\gamma b \in I$. In other words $I\Gamma S \subseteq I$. Next I is called a right– Ideal of S if $S\Gamma I \subseteq I$. It is called Γ – the ideal of S if $S\Gamma I \subseteq I$ and $I\Gamma S \subseteq I$. I is called bi – Γ – the ideal of S if $I\Gamma S\Gamma I \subseteq I$. [6]

$A \subseteq S$ is called $(m,n) - \Gamma$ – ideal if $A^m \Gamma S \Gamma A^n \subseteq A$ where m and n are non-negative integers. In this case, A^0 is defined with $A^0 \Gamma S = S$ and $S \Gamma A^0 = S$ [5].

3. Γ–Semigroup generated by a semigroup

In this article, Γ is a binary operation set on S such that Γ – semigroup is redefined as follows.

Definition 3.1 Let S be a non-empty set and Γ be a set of associative binary operations on S. S is called Γ – semigroup if $(x\alpha y)\beta z = x\alpha(y\beta z)$ for every $x, y, z \in S$ and for every $\alpha, \beta \in \Gamma$.

Note that, since Γ it is a collection of associative binary operations on S, it means for every $x, y \in S$ and for every $\alpha \in \Gamma$ satisfies $x\alpha y \in S$. Definition 3.1 no need to be written $x\alpha y \in S$ for every $x, y \in S$ and for every $\alpha \in \Gamma$ .All the classical ideas developed in the Γ – Semigroup defined by Sen [1] also apply to Γ – semigroup defined in this article. Now, we define the equality of two binary operations on a set.

Definition 3.2 Two binary operations α and β on the non-empty set S are said equally in S if $x\alpha y = x\beta y$ for every $x, y \in S$.

The next discussions show that every element of semigroup S can define a binary operation on S. In this article it Γ is the set of all binary operations defined of the elements of S, then S is a Γ – semigroup. Now, we define a binary operation from the elements of a semigroup.

Definition 3.3 Let S be a non-empty set, α be a binary operation on S and (S, α) be a semigroup if $a \in S$ defined binary operation αa on S with $x\alpha a\alpha y = (x\alpha a)\alpha y = x\alpha(a\alpha y)$ for every $x, y \in S$. Next, $\alpha a\alpha$ it can be written as α_a .

Example 3.4 Let (S, α) be a semigroup with $S = \{a, b, c\}$ dan α are given in the following Caley table.

α	a	b	c
a	a	b	c
b	b	b	b
c	c	c	c

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For $b \in S$, the operation of binary α_b is defined with $x\alpha_b y = (x\alpha b)\alpha y$ for every $x, y \in S$ such that the following Caley table is obtained.

α_b	a	b	c
a	b	b	b
b	b	b	b
c	c	c	c

From definition 3.3 obtained some of the following simple properties.

Theorem 3.5 If *e* is an identity element or left identity or right identity on semigroup (S,α) then $\alpha_e = \alpha$.

Proof

For all $x, y \in S$, $x\alpha_e y = (x\alpha_e)\alpha y = x\alpha y$.

By definition 3.3, its yields $\alpha_e = \alpha$. ■

Theorem 3.6 Let (S,α) is a semigroup and $a, b \in S$. If $a\alpha x = b\alpha x$ or $x\alpha a = x\alpha b$ for every $x \in S$ then $\alpha_a = \alpha_b$.

Proof

For all $x, y \in S$,

$$\begin{aligned} x\alpha_a y &= (x\alpha_a)\alpha y \\ &= (x\alpha_a)\alpha y \\ &= x\alpha_b y \end{aligned}$$ ■

Example 3.7 Let (S,α) be a Semigroup with $S = \{a,b,c,d\}$ and α defined by the Caley table below.

α	a	b	c	d
a	a	a	a	a
b	a	b	c	d
c	a	a	a	a
d	a	b	c	d

Clearly, $\alpha_b = \alpha_d = \alpha$ and $\alpha_a = \alpha_c$.

$\alpha_b = \alpha_d$	a	b	c	d
a	a	a	a	a
b	a	b	c	d
c	a	a	a	a
d	a	b	c	d

$\alpha_a = \alpha_c$	a	b	c	d
a	a	a	a	a
b	a	a	a	a
c	a	a	a	a
d	a	a	a	a

This theorem show if (S,α) is a semigroup and $\Gamma = \{\alpha_a \mid a \in S\}$ with α_a as defined on the Definition 3.3 then *S* is a Γ – semigroup.

Theorem 3.8 It (S,α) is a semigroup and $\Gamma = \{\alpha_a \mid a \in S\}$ then *S* is a Γ – semigroup.

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Proof: Let (S,α) be a semigroup and $\Gamma=\{\alpha_a \mid a\in S\}$. Take an arbitrary $x,y,z\in S$ and $\alpha_a\in\Gamma$.

$$\begin{aligned}(x\alpha_a y)\alpha_a z &= ((x\alpha a)\alpha y)\alpha(a\alpha z)\\ &= (x\alpha a)\alpha(y\alpha(a\alpha z))\\ &= x\alpha_a(y\alpha_a z)\end{aligned}$$

So for every $\alpha_a\in\Gamma$ is an associative binary operation on *S*. It follows that for every $x,y,z\in S$ and $\alpha_a,\alpha_b\in\Gamma$ satisfies

$$\begin{aligned}(x\alpha_a y)\alpha_b z &= ((x\alpha a)\alpha y)\alpha(b\alpha z)\\ &= (x\alpha a)\alpha(y\alpha(b\alpha z))\\ &= x\alpha_a(y\alpha_b z)\end{aligned}$$

Finally, *S* is a Γ – semigroup.■

Example 3.9 From Example 3.4, $S=\{a,b,c\}$

α	a	b	c
a	a	b	c
b	b	b	b
c	c	c	c

We obtain $\Gamma=\{\alpha_a,\alpha_b,\alpha_c\}$ with $\alpha_a=\alpha$ and α_b,α_c provided by the table below.

α_b	a	b	c
a	b	b	b
b	b	b	b
c	c	c	c

α_c	a	b	c
a	c	c	c
b	b	b	b
c	c	c	c

The following theorems show the properties of Γ – semigroup derived from it's generating semigroup.

Theorem 3.10 If (S,α) is a commutative semigroup and $\Gamma=\{\alpha_a \mid a\in S\}$ then *S* is a commutative Γ – semigroup.

Proof

Take an arbitrary $x,y\in S$ and $\alpha_a\in\Gamma$.

$$\begin{aligned}x\alpha_a y &= (x\alpha a)\alpha y\\ &= y\alpha(x\alpha a)\\ &= y\alpha(a\alpha x)\\ &= y\alpha_a x\end{aligned}$$

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Theorem 3.11. If (S,α) it is a (m,n) – regular semigroup and $\Gamma=\{\alpha_a \mid a\in S\}$ with *m*,*n* are positive integers then *S* is a (m,n) – regular Γ – semigroup.

Proof :Let (S,α) be a (m,n) – regular semigroup. For $a\in S$, a^m it is written as $a^m = a\alpha a\alpha a\alpha\ldots.\alpha a$ (the number of *a* is *m* times). For every, $a\in S$ there exist $x\in S$ such that

$$
\begin{aligned}
a &= a^m\alpha x\alpha a^n\\
&= \left(a^m\alpha x\alpha a^n\right)^m \alpha x\alpha\left(a^m\alpha x\alpha^n\right)^n\\
&= a\alpha\left(a^{m-1}\alpha x\alpha a^n\right)\alpha a\alpha\left(a^{m-1}\alpha x\alpha a^n\right)\alpha\ldots\ldots a\alpha\left(a^{m-1}\alpha x\alpha a^n\right)\alpha x\alpha\\
&= \left(a^m\alpha x\alpha a^{n-1}\right)\alpha a\alpha\left(a^m\alpha x\alpha a^{n-1}\right)\alpha a\ldots\ldots..\alpha\left(a^m\alpha x\alpha a^{n-1}\right)\alpha a\\
&= a\alpha a^{m-1}\alpha x\alpha a^n a\alpha a^{m-1}\alpha x\alpha a^n\ldots\ldots.\alpha a^{m-1}\alpha x\alpha a^n\alpha x\alpha a^m\alpha x\alpha a^{n-1}\alpha a.\\
&\quad\ldots..\alpha a^m\alpha x\alpha a^{n-1}a\\
&\in \underbrace{a\Gamma a\Gamma\ldots.\Gamma a}_{a\,is\,m\,times}\Gamma x\Gamma\underbrace{a\Gamma\ldots.\Gamma a}_{a\,is\,n\,times}\\
&= a^m\Gamma x\Gamma a^n
\end{aligned}
$$

Finally, *S* is a (m,n) – regular Γ – semigroup. ■

Theorem 3.12 If (S,α) is a regular semigroup and $\Gamma=\{\alpha_a \mid a\in S\}$ then *S* is a completely regular Γ – Semigroup.

Proof :Let (S,α) is a regular Semigroup. Take an arbitrary $a\in S$, then there exists $b\in S$ such that

$$
\begin{aligned}
a &= (a\alpha b)\alpha a\\
&= \left((a\alpha b)\alpha a\right)\alpha b\alpha\left(a\alpha(b\alpha a)\right)\\
&= (a\alpha b)\alpha\left((a\alpha b)\alpha a\right)\alpha(b\alpha a)\\
&= (a\alpha b)\alpha a\alpha(b\alpha a)\\
&= a\alpha_b a\alpha_b a
\end{aligned}
$$

and

$$a\alpha_b a = a\alpha a$$

It follows that for every $a\in S$,there exists $\alpha\in\Gamma$ and $b\in S$ such that $a=(a\alpha_b a)\alpha_b a$ and $a\alpha_b a = a\alpha a$. Finally, *S* is a completely regular Γ – semigroup. ■

Theorem 3.13 If *A* is a left ideal of semigroup (S,α) and $\Gamma=\{\alpha_a \mid a\in S\}$ then *A* is left Γ – ideal of Γ – semigroup *S*.

Proof: Let *A* be a left ideal of the semigroup (S,α). Take an arbitrary $x \in S$, $\alpha r \in \Gamma$ and $a \in A$. It follows that $x\alpha_r a = (x\alpha r)\alpha a \in A$. If *A* is the ideal or right ideal of semigroup (S,α) and $\Gamma = \{\alpha_a \mid a \in S\}$ then *A* is a Γ – ideal or right Γ – ideal respectively of Γ – semigroup *S*.

Theorem 3.14. If *A* be a bi – Γ– ideals in Γ– semigroup (S,α) and $\Gamma = \{\alpha_a \mid a \in S\}$ then *A* is bi – Γ – ideal of Γ – semigroup *S*.

Proof :Let *A* be a bi – Γ– ideals in Γ– semigroup (S,α) then $A\alpha S\alpha A \subseteq A$. Take an arbitrary $a,b \in A, x \in S$ and $\alpha_r, \alpha_s \in \Gamma$. It follows that

$$
\begin{aligned}
a\alpha_r x\alpha_s b &= (a\alpha r)\alpha x\alpha(s\alpha b)\\
&= a\alpha((r\alpha x)\alpha s)\alpha b\\
&\in A\alpha S\alpha A \subseteq A
\end{aligned}
$$

Clearly, *A* is a Bi – Γ– ideals in Γ– semigroup.

4. Conclusion

Every element of a semigroup could define the binary operations on the semigroup. A collection of all binary operations defined by the element of semigroup could generate Γ– semigroup of the semigroup. Properties of a Γ– semigroup follow of it's generating semigroup. For the future, Γ– semigroup will generate another Γ– semigroup which called dual Γ– semigroup.

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