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Nama : Drs. Bayu Surarso, M.Sc Ph.D
NIP. 19631105 198803 1 001
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Bidang Ilmu: Matematika

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The distance between two convex sets in Hilbert space

Sagala, Yuri C. ; Hariyanto, Susilo ; Sumanto Y.D.; Udjiani, Titi
Save all to author list

[a] Department of Mathematics, Universitas Diponegoro, Semarang, Jawa Tengah, 50275, Indonesia

Abstract
In this paper we will discuss how to determine the distance between two convex sets in Hilbert space. This problem came from measuring the shortest distance between two cities, which consider by determining the distance between two buildings in each city. In this problem, the cities are considered as the sets and the buildings are points. Furthermore, based on this problem, it is generalized to determine the distance between two convex sets in Hilbert space that solved by optimization concept by measuring maximal distance between two parallel supporting hyperplanes that separate them. Therefore, it is given some example to understanding, such as the distance between two normed balls, ellipsoids, and linear varieties. © 2021 Author(s).

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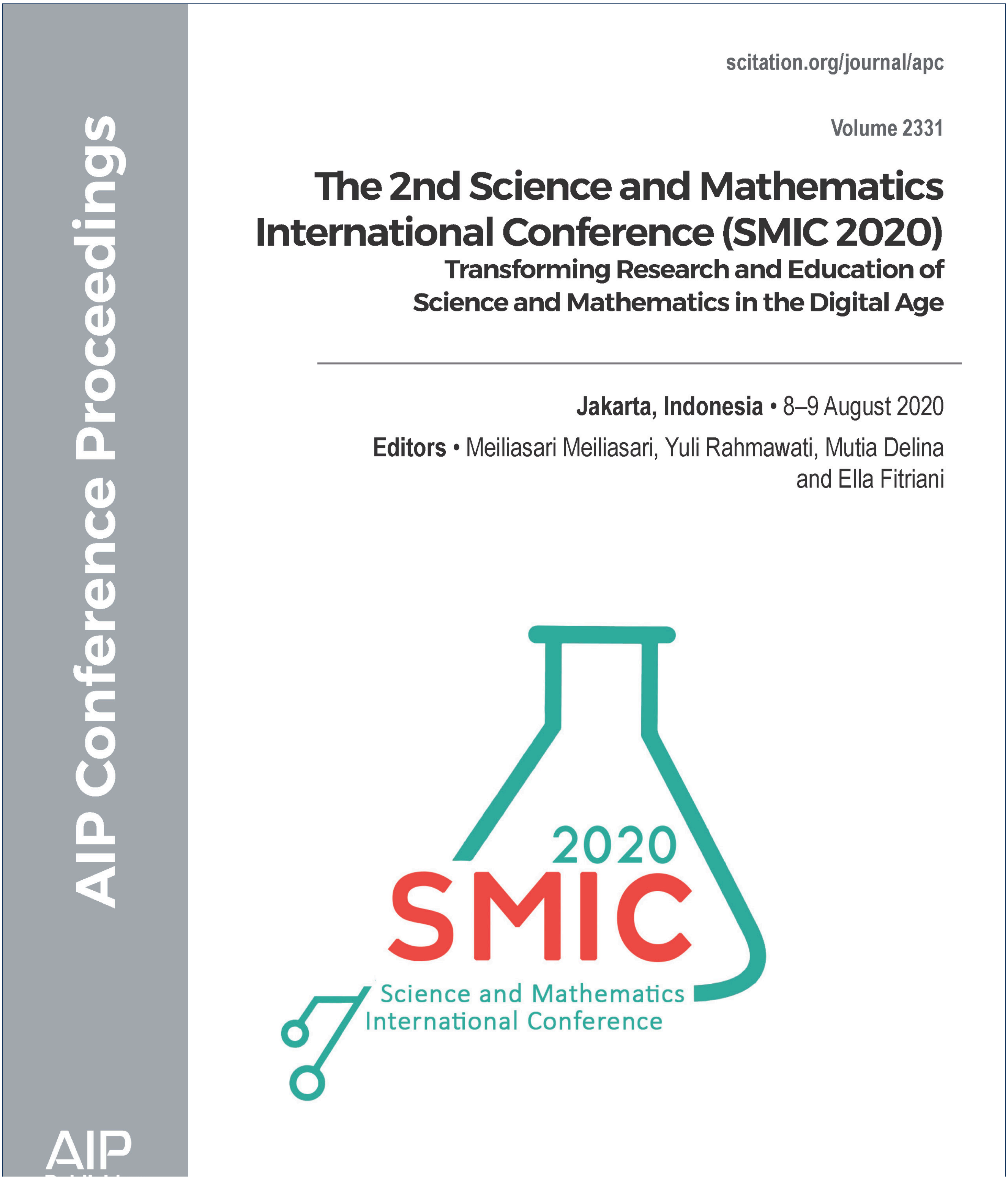
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‘Counting up’ Strategy as an Alternative to Solve Subtraction Problems with Regrouping

Puspita Sari[1, 2, a)] and Dindyal Jaguthsing[2, b)]

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Abstract. This paper intends to explore how the ‘counting up’ strategy on an empty number line could help children overcome their mistakes in performing column subtraction problems that require regrouping. Solving subtraction problems with regrouping in column strategy is often frustrating for most children because it involves regrouping numbers in place value. In this qualitative case study, how two second graders develop their thinking while engaging in a designed instructional activity were observed and analyzed. The data collection included observations, conversational interviews, and children’s written works. The results showed that the designed activity with a measurement context and an empty number line model is found to be helpful for children to develop the idea of ‘counting up’ in solving subtraction problems that require regrouping. The children gradually develop their counting up strategy from jumps of ones to jumps of tens to simplify the use of the empty number line model. The results help us understand that we must provide a meaningful support for children to explore various calculation strategies to improve their flexibility in solving subtraction problems.

INTRODUCTION

Inability in handling numbers flexibly in daily life situations has been a major concern for decades. When children start their primary school, they are introduced to the number sequence and counting of objects one by one, followed by addition and subtraction of single-digit numbers by counting on and counting back strategies using concrete materials or their fingers. Once they learn place value, teachers usually show them how to perform the column strategy or algorithm for solving addition and subtraction of two-digit or multi-digit numbers. Previous studies [1-3] agreed that the calculations become more problematic for children as they start regrouping numbers in solving addition and subtraction problems. Therefore, introducing the algorithm strategy prematurely without context problems and sufficient understanding of numbers can cause an overreliance on the use of algorithm and issues with addition and subtraction as a consequence [4, 5].

Based on literatures [6, 7], the operation of addition can be classified into two types, i.e. the aggregation (finding a total of two quantities) and the augmentation (finding an increased value), whereas the operation of subtraction has four different structures, i.e. partitioning (e.g. “how many are left?”), reduction (e.g. “what is the reduced price?”), comparison (e.g. “what is the difference?”), and inverse-of-addition (e.g. “how many more needed?”). The National Council of Teachers of Mathematics [8] proposed that students should have a meaningful understanding of numbers and operations as well as use efficient and accurate methods for computing, either using mental strategies or an algorithm. Apart from the strategy they use, students should be able to explain their strategies and be flexible in choosing the best strategy that fits for certain numbers. For example, the counting back strategy is considered more efficient to solve subtraction of “91 – 3”, but to find the difference of “91 – 89”, one might think that the counting up is a better method. While the algorithm is best to solve “56789 – 12345”, but to find the amount of money left after spending Rp29,850 and paying with a Rp50,000 cash (Rp = *Rupiah*) will suggest a different method than algorithm. Thus, we need to provide children with various situations to explore meaningful strategies in performing arithmetic operations.

Studies associated with counting strategies up to 20 [9-13] revealed children’s informal strategies and the utilised number facts in solving additions and subtractions. Other studies associated with counting multidigit numbers [1, 7, 14-17] indicated the use of models and contexts to help children acquire the place value concept as well as the number structure to lay the foundation for more formal strategies, including mental arithmetic strategies. Fuson *et al*

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Assessing Metacognitive Beliefs among Science Education Students based on The Metacognition Questionnaire-30 (MCQ-30)

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Abstract. Students always feel about self-awareness on their cognitive functions as a part of metacognitive beliefs. This aspect is being important to support learning process consistently. The aim of this case study was assessed students' metacognitive beliefs during learning science. The Metacognition Questionnaire-30 (MCQ-30) which is being instrument for this study, consists of five subscales namely positive beliefs about worry, negative beliefs about uncontrollability and danger of worry, cognitive confidence, need to control thoughts, and cognitive self-consciousness has been developed originally by Wells and Cartwright-Hatton (2004). The questionnaire (30 items) which rated on a 4-point Likert scale was administered to 102 students from science education study program at public university in Bandung, Indonesia. This research subject come outs from all students in one study program for all batches. Rasch Model shows that Person Reliability is 0.71 and Item Reliability is 0.98, therefore the MCQ-30 fitted to use for university students. Results describe that the highest response is students trust their memory while the lowest response is they are not going to punish themselves if they are not controling their thoughts. Lastly, how to manage the students' memory during learning science is to be implication for further research.

INTRODUCTION

Metacognitive ability gives an opportunity how students do correction self-performance during learning process. This strategy can be diverse for each student based on their experiences and practices. Furthermore, enhancing the metacognition will generate widespread benefits for education field. For instance, students can control the behavior effectively as well as monitor their performance [1]. Due to the metacognitive growth which more effective and stronger, young age generation is the right time for improving metacognitive. As consequently, self-control will manage better [2]. This condition gives the students a chance to builld metacognitive awareness during learning science and how students identify and manage in line with educational goal. This study was elaborated the students' metacognition beliefs based on The Metacognition Questionnaire-30 (MCQ-30) among science education students. This research is very important to serve students in exploring their metacognition in science learning. The aim of this case study was assessed students' metacognitive beliefs during learning science.

Metacognition, a strategy to reflect, monitor, and regulate the cognition or mind, was first initiated by Flavell in 1979 [2]. There are some versions of domain or aspect for explaining the meta-cognitive, such as knowledge of cognition dimension and regulation of cognition dimension [3]. Whereas this study uses The Metacognition Questionnaire-30 (MCQ-30) for assessing metacognitive beliefs which consists of five subscales [4, 5]. This

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Solving Scattering Problems Using the Stabilized Finite Element and the Algebraic Multigrid

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Abstract. Electromagnetic waves are waves that propagate without the need of medium. One of the equations to describe the phenomenon of electromagnetic wave scattering problems is the Helmholtz equation. There are difficulties in solving the Helmholtz equation numerically, two of which that the solution form is highly oscillatory and the numerical solution of the high-frequency cases has low accuracy, which is called "pollution effects". Due to the limitation of memory and CPU size in a digital computer, simulating this problem with a large size of computational points is impossible. Besides, solving the linear system of the Helmholtz equation is quite challenging. It is difficult to find an efficient iterative solver that can converge with only a few iterations. This research presents a numerical scheme to minimize the "pollution effect" that appear in the numerical solution of the Helmholtz equation. The numerical scheme gives better accuracy than the standard scheme of the finite element. Moreover, we solve the linear system by using an algebraic multigrid as a preconditioner in a main iteration routine, which is based on a preconditioned variant of generalized conjugate residual (GCR) methods. This solver can give a better performance regarding its number of iteration and the time cost.

INTRODUCTION

The Helmholtz equation is a mathematical model commonly used for describing many physical phenomena involving time-harmonic wave propagation, such as acoustics, electrodynamics, and electromagnetic radiation. Physicists and engineers are interested in the robust and efficient numerical simulation process of these phenomena for many decades. However, numerically solving the Helmholtz equation poses many challenges from different aspects. First, as the frequency of wave increases, the accuracy of the solution obtained using some standard numerical schemes, for example, a linear Galerkin finite element method deteriorates dramatically because the phase velocity of the numerical wave differs from the physical wave. This lack of robustness is called the "pollution effect." Possible available remedies in the literature include the modification of the original classical scheme by adding the perturbation term involving the stability parameter and the residual term, e.g., stabilized Galerkin/least squared finite element method [1,2,3]. The other possibility is to enrich the polynomial-type finite element space by adding the bubble function space, e.g., residual-free bubble method [4, 5].

Second, at high-frequency, the Helmholtz equation's solution is highly oscillatory; more discretization points are needed to satisfy the points per wavelength (PPW) requirement, causing the problem size becomes more extensive. It is quite costly to solve such problems. Moreover, classical iterative methods without preconditioning often fail to converge. Some classical iterative methods, multigrid, and preconditioners were tested by Ernst and Gander [6]. They elaborated on reasons why classical iterative methods fail. For instance, in the convergence of Krylov methods, the efficient transport of information in the preconditioner is important to solving such problems. However, an incomplete lower/upper (ILU) factorization with no fill-in preconditioner does not effectively bring this transport. Even the stronger preconditioners, such as ILU with threshold (ILUT(1e-2)) factorization, does not suffice either. Ernst and Gander also discovered why the standard multigrid fails as a solver--solving Helmholtz problems at high-frequency numerically, causing the high-frequency error. This error is particularly introduced by

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An Explanation of Sedna Orbit from Condensed Matter or Superconductor Model of the Solar System: A New Perspective of TNOs

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Abstract. In a recent paper, we argued in favor of the Gross-Pitaevskii model as a complete depiction of both the close planetary system and winding worlds, particularly considering the idea of chirality and vortices in universes. In this paper, we apply the new model based on Bogoliubov-de Gennes equation correspondence with Bohr-Sommerfeld quantization rules. Then we put forth an argument that from Bohr-Sommerfeld quantization rules, we can come up with a model of quantized orbits of planets in our solar system, be it for inner planets and also for Jovian planets. In effect, we also tried to explain Sedna's orbit in the same scheme.

INTRODUCTION

A few abbreviations used in this paper: TNO: trans-Neptunian object; KBO: Kuiper-Belt Object. Every once in a while, cosmology and astronomy revelations have opened our eyes that the universe is substantially more entangled than what it appeared in 100-200 years prior. What's more, regardless of all invading fame of General Relativistic augmentation to Cosmology, considering antiquated Greek rationalists' theories, for example, hydor model (Thales) and streaming liquid model (Heracleitus) it appears to be as yet qualified to ask: does it imply that the Ultimate hypothesis that we attempt to discover ought to compare to hydrodynamics or a disturbance hypothesis [1-3].

Meanwhile, in a recent article, we presented some new contentions on the hypothetical small star thought to be an ally to our Sun, known as the Nemesis, which is proposed to clarify an apparent pattern of mass eradications in Earth's history. Some guessed that such a star could influence the circle of articles in the far external close planetary system, sending them on a crash course with Earth. While ongoing cosmic reviews neglected to discover any proof that such a star exists, we layout in this article some hypothetical discoveries including our own, suggesting that such a dwarf star companion of the Sun remains a possibility [4]. And one good indicator for such a dwarf companion of the Sun is Sedna, a planetoid which has been discovered around 2004 by Mike Brown and his Caltech team. Sedna location and eccentric orbit are such that it is not supposed to be there [5-10].

Therefore a physical explanation of why Sedna is located there can be a good start to begin to search the existence and location of the supposedly dwarf companion of the Sun.

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The Distance between Two Convex Sets in Hilbert Space

by Susilo Hariyanto

The Distance between Two Convex Sets in Hilbert Space

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Abstract. In this paper we will discuss how to determine the distance between two convex sets in Hilbert space. This problem came from measuring the shortest distance between two cities, which consider by determining the distance between two buildings in each city. In this problem, the cities are considered as the sets and the buildings are points. Furthermore, based on this problem, it is generalized to determine the distance between two convex sets in Hilbert space that solved by optimization concept by measuring maximal distance between two parallel supporting hyperplanes that separate them. Therefore, it is given some example to understanding, such as the distance between two normed balls, ellipsoids, and linear varieties.

INTRODUCTION

Let $\mathcal{Y}$ be a city and let $\boldsymbol{z}$ be some object outside $\mathcal{Y}$. City $\mathcal{Y}$ has a lot of buildings, such as markets, schools, hospitals, government offices, and others. We can measure the distance between the city $\mathcal{Y}$ and $\boldsymbol{z}$ by determining the nearest building(s) in city $\mathcal{Y}$ to $\boldsymbol{z}$. This problem is called as best approximation problem. The related theories in this problem are Approximation Theory, Functional Analysis, Convex Analysis, Optimization, Linear Algebra, and others. If the city is declared as the set, the buildings are the points, and $\boldsymbol{z} \in \mathcal{Z}$, we will compute the distance between the sets $\mathcal{Y}$ and $\mathcal{Z}$ by finding the nearest point between them, such that $\|\boldsymbol{y} - \boldsymbol{z}\|$ attains the minimum value, where $\boldsymbol{y} \in \mathcal{Y}$ and $\boldsymbol{z} \in \mathcal{Z}$.

Best approximation problem was introduced in ref. [1] who proposed Minimum Norm Duality (MND) theorem to determine the distance between a convex set and a point outside it. This theorem said that the shortest distance between a convex set $\mathcal{Y}$ and a point $\boldsymbol{z} \notin \mathcal{Y}$ is equal to the maximum distance between a point $\boldsymbol{z} \notin \mathcal{Y}$ to any separating hyperplane of $\boldsymbol{z}$ and $\mathcal{Y}$ (see fig. FIGURE **2**). Next, ref. [2] computed the distance between two convex polygons by computational complexity. Furthermore, ref. [3] studied minimum distance to the complement of a convex set, and ref. [4] modified MND theorem that proposed by [1] to compute the distance between two convex sets.

Based on these studies, it is shown how to compute the distance between two sets by measuring the distance between two parallel hyperplanes that separate them. Some restriction that applied in this paper is

1. The solution of best approximation is unique, so the sets must be convex, [5,6]. Since we compute the minimum distance by measuring the distance between two parallel hyperplane that separate them, the existence of separating hyperplane cannot be guaranteed if one of the set is not convex, see fig. FIGURE **1**.
2. Some of studies [2–4,7] have considered the minimum distance problem in Euclidean space $\mathbb{R}^n$, although Luenberger [1] and Deutsch [5] has defined this problem in inner product space (or Hilbert space). Based on fact that hyperplane is defined on inner product space, we will consider this problem in larger space, that Hilbert space.

The plan of this paper is as follows. In section 1 contains necessary background. Section 2 explains the basic facts on norms. In section 3, it is shown some assertions of hyperplane, and section 4 shows the theories about the best approximation, and we give some examples in section 5.

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NORMS

As a basic of minimum distance, first we show the definition of norm in definition 1 below.

Definition 1 [6] *Norm* $\|\cdot\|$ *is a mapping from linear space to non-negative real number such that the following properties are satisfied:*

1. $\|\boldsymbol{u}\| \geq 0$ (positivity)
2. $\|\alpha \boldsymbol{u}\| = |\alpha|\|\boldsymbol{u}\|$ (homogeneity)
3. $\|\boldsymbol{u}+\boldsymbol{v}\| \leq \|\boldsymbol{u}\| + \|\boldsymbol{v}\|$ (triangle inequality)

For all $\boldsymbol{u}$ and $\boldsymbol{v}$ are vectors in the linear space, and $\alpha \in \mathbb{R}$.

Note that norm is a convex function. The dual norm of $\|\cdot\|$ is denoted by $\|\cdot\|'$, that obtained by this following way. Given a vector $\boldsymbol{w}$ in a linear space, then $\|\boldsymbol{w}\|' = \max_{\|\boldsymbol{u}\| \leq 1} \langle \boldsymbol{u}, \boldsymbol{w} \rangle$, and the Hölder inequality states that $|\langle \boldsymbol{u}, \boldsymbol{w} \rangle| \leq \|\boldsymbol{u}\|\|\boldsymbol{w}\|'$. If $|\langle \boldsymbol{u}, \boldsymbol{w} \rangle| = \|\boldsymbol{u}\|\|\boldsymbol{w}\|'$ then it is aligned with respect to $\|\cdot\|$ or $\|\cdot\|'$.

One example of dual norms is related to the $L_p[a,b]$ norm, which contains all integrated function in $[a,b]$ that defined as

$$\|u\|_p = \left(\int_a^b |u(t)|^p dt \right)^{\frac{1}{p}}$$

Where $1 < p < \infty$. The dual norm is $L_q[a,b]$ norm

$$\|w\|_q = \left(\int_a^b |w(t)|^q dt \right)^{\frac{1}{q}}$$

where $\frac{1}{p} + \frac{1}{q} = 1$. If $p = 2$, the dual norm is itself, and L_2-space is contained in Hilbert space.

HYPERPLANE

According to ref. [8,9], hyperplane that denoted by $\mathcal{H}$, is defined in inner product space or Hilbert space as a set of vectors $\boldsymbol{x}$ which satisfies $\mathcal{H} = \{\boldsymbol{x} | \langle \boldsymbol{a}, \boldsymbol{x} \rangle = \alpha\}$. Hyperplane divide a linear space into two half spaces such that

$$\mathcal{H}^{\leq} = \{\boldsymbol{x} | \langle \boldsymbol{a}, \boldsymbol{x} \rangle \leq \alpha\}$$
$$\mathcal{H}^{\geq} = \{\boldsymbol{x} | \langle \boldsymbol{a}, \boldsymbol{x} \rangle \geq \alpha\},$$

If the boundary line is excluded, then we have the two open half spaces

$$\mathcal{H}^{<} = \{\boldsymbol{x} | \langle \boldsymbol{a}, \boldsymbol{x} \rangle < \alpha\}$$
$$\mathcal{H}^{>} = \{\boldsymbol{x} | \langle \boldsymbol{a}, \boldsymbol{x} \rangle > \alpha\}.$$

In other words, half spaces $\mathcal{H}^{<}$ and $\mathcal{H}^{>}$ are separated by hyperplane $\mathcal{H}$, or $\mathcal{H}$ is a called as **separating hyperplane**. This below theorem guarantees the existence of separating hyperplane.

Theorem 2 [10] (Separating Hyperplane Theorem) *Let* $\mathcal{Y}$ *and* $\mathcal{Z}$ *be nonempty disjoint convex sets. Then there exist* $\boldsymbol{a} \neq \boldsymbol{0}$ *and* $\alpha \in \mathbb{R}$ *such that* $\langle \boldsymbol{a}, \boldsymbol{x} \rangle - \alpha \leq 0, \forall x \in \mathcal{Y}$ *and* $\langle \boldsymbol{a}, \boldsymbol{x} \rangle - \alpha \geq 0, \forall \boldsymbol{x} \in \mathcal{Z}$.

It is equivalent to say that separating hyperplane $\mathcal{H}$ always exist if both $\mathcal{Y}$ and $\mathcal{Z}$ are convex sets. But if either $\mathcal{Y}$ or $\mathcal{Z}$ is not convex, theorem 2 fails, as shown on fig. 1.

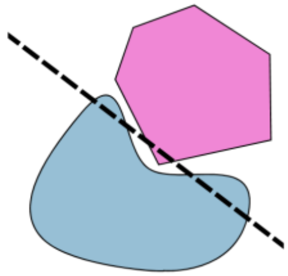

FIGURE 1. Separating theorem does not apply if one of the bodies is not convex

If hyperplane $\mathcal{H}$ is tangent to a set, then $\mathcal{H}$ is **supporting hyperplane**, as shown on definition 3.

Definition 3 [4] *Let $\mathcal{Y}$ be a convex set. Hyperplane $\mathcal{H} = \{\boldsymbol{x}|\langle \boldsymbol{a}, \boldsymbol{x}\rangle = \alpha\}$ is said to be a supporting hyperplane of $\mathcal{Y}$ if*

$$\sup_{\boldsymbol{x} \in \mathcal{Y}} \langle \boldsymbol{a}, \boldsymbol{x}\rangle = \alpha.$$

Since $x \in \mathcal{H}$, definition 3 also means that x and convex set $\mathcal{Y}$ are separated by hyperplane $\mathcal{H}$, and hyperplane $\mathcal{H}$ supports $\mathcal{Y}$ at $\boldsymbol{x}^*$. The first step to measure the distance between two convex sets is choosing a pair of parallel separating hyperplanes that support each convex sets. In the next section, it is shown how to compute the distance between two convex sets through the dual problem.

THE DISTANCE BETWEEN TWO CONVEX SETS

Let $\|\cdot\|$ be some arbitrary norm and $\|\cdot\|'$ the corresponding dual norm. Let $\mathcal{Y}$ and $\mathcal{Z}$ be two nonempty disjoint convex sets. The distance between two convex sets $\mathcal{Y}$ and $\mathcal{Z}$ is defined as

$$\operatorname{dist}(\mathcal{Y}, \mathcal{Z}) = \inf\{\|\boldsymbol{y} - \boldsymbol{z}\| \,|\, \boldsymbol{y} \in \mathcal{Y}, \boldsymbol{z} \in \mathcal{Z}\} \quad (1)$$

According to ref. [11] and [10], hyperplane is an example of convex sets. In $\mathbb{R}^n$ space, hyperplane can be visualized as flat. To compute the distance between two convex sets by its dual, we must construct the supporting hyperplanes. So, in this section we will proof that the minimum distance between two convex sets is equal to the maximum distance between two separating hyperplanes.

First, we consider the distance between two parallel hyperplanes. Let $\mathcal{H} = \{\boldsymbol{x}|\langle \boldsymbol{a}, \boldsymbol{x}\rangle = \alpha\}$ be a hyperplane, and $\boldsymbol{z} \notin \mathcal{H}$. By equation (1), the distance between $\mathbf{z}$ and $\mathcal{H}$ is

$$\operatorname{dist}(\mathbf{z}, \mathcal{H}) = \inf_{\mathbf{x} \in \mathcal{H}} \|\mathbf{z} - \mathbf{x}\|.$$

Ref. [7] has proved the distance between point and hyperplane is

$$\operatorname{dist}(\mathbf{z}, \mathcal{H}) = \left|\frac{\langle \boldsymbol{a}, \boldsymbol{z}\rangle - \alpha}{\|\boldsymbol{a}\|'}\right| \quad (2)$$

Since $\boldsymbol{z} \notin \mathcal{H}$, we must choose two disjoint hyperplanes $\mathcal{H}_1 = \{\boldsymbol{x}|\langle \boldsymbol{a}, \boldsymbol{x}\rangle = \alpha_1\}$ and $\mathcal{H}_2 = \{\boldsymbol{x}|\langle \boldsymbol{a}, \boldsymbol{x}\rangle = \alpha_2\}$. The distance between them is defined as

$$\operatorname{dist}(\mathcal{H}_1, \mathcal{H}_2) = \inf\{\|\boldsymbol{x_1} - \boldsymbol{x_2}\| \,|\, \boldsymbol{x_1} \in \mathcal{H}_1, \boldsymbol{x_2} \in \mathcal{H}_2\} \quad (3)$$

By (2), we obtain

$$\operatorname{dist}(\mathcal{H}_1, \mathcal{H}_2) = \left|\frac{\langle \boldsymbol{a}, \boldsymbol{x_1}\rangle - \alpha_2}{\|\boldsymbol{a}\|'}\right|, \qquad \forall \boldsymbol{x_1} \in \mathcal{H}_1$$

Therefore, since $\langle \boldsymbol{a}, \boldsymbol{x_1}\rangle = \alpha_1$, $\forall \boldsymbol{x_1} \in \mathcal{H}_1$,

$$\operatorname{dist}(\mathcal{H}_1, \mathcal{H}_2) = \left|\frac{\alpha_1 - \alpha_2}{\|\boldsymbol{a}\|'}\right| \quad (4)$$

Look again eq. (1). Let $\mathcal{Y} - \mathcal{Z} = \mathcal{X}$, and $\mathbf{0} \notin \mathcal{X}$. The distance between $\mathcal{Y}$ and $\mathcal{Z}$ is equal to

$$\operatorname{dist}(\mathcal{Y}, \mathcal{Z}) = \operatorname{dist}(\mathbf{0}, \mathcal{X}) = \inf_{\boldsymbol{x} \in \mathcal{X}} \|\boldsymbol{x}\|$$

Since $\mathcal{Y}$ and $\mathcal{Z}$ are convex, $\mathcal{X}$ is also convex.

Lemma 4 [4] (The Least Norm Problem) *There exists* ***a*** *point* $\boldsymbol{x}^* \in \boldsymbol{\mathcal{X}}$ *such that* $\|\boldsymbol{x}^*\| \leq \|\boldsymbol{x}\|, \forall \boldsymbol{x} \in \boldsymbol{\mathcal{X}}$*.*

Lemma 4 expresses an optimization problem

$$\begin{aligned} &\text{minimize} \quad \|\boldsymbol{x}\| \\ &\text{subject to } \boldsymbol{x} \in \mathcal{X} \end{aligned} \tag{5}$$

is always solvable, that attains a minimizer $\boldsymbol{x}^* \in \mathcal{X}$. But, if $\|\cdot\|$ is a strictly convex norm, $\boldsymbol{x}^*$ is unique.

Now, let's construct the dual problem of (5). For this purpose, we show the support functions

$$\begin{aligned} \alpha(\boldsymbol{a}) &= \sup_{\boldsymbol{y} \in \mathcal{Y}} \langle \boldsymbol{a}, \boldsymbol{y} \rangle ; \\ \beta(\boldsymbol{a}) &= \inf_{\boldsymbol{z} \in \mathcal{Z}} \langle \boldsymbol{a}, \boldsymbol{z} \rangle \end{aligned} \tag{6}$$

which are well defined for any vector $\boldsymbol{a}$. If any $\boldsymbol{a} \neq \boldsymbol{0}$ and $\alpha(\boldsymbol{a}) \leq \beta(\boldsymbol{a})$ then the parallel hyperplanes

$$\mathcal{H}_\alpha = \{\boldsymbol{x} | \langle \boldsymbol{a}, \boldsymbol{x} \rangle = \alpha(\boldsymbol{a})\}, \ \mathcal{H}_\beta = \{\boldsymbol{x} | \langle \boldsymbol{a}, \boldsymbol{x} \rangle = \beta(\boldsymbol{a})\}$$

Separates $\mathcal{Y}$ and $\mathcal{Z}$. By (4),

$$\text{dist}(\mathcal{H}_\alpha, \mathcal{H}_\beta) = \frac{\beta(\boldsymbol{a}) - \alpha(\boldsymbol{a})}{\|\boldsymbol{a}\|'}.$$

In figure below, it is shown how to choose the hyperplane for calculating the distance of two convex sets.

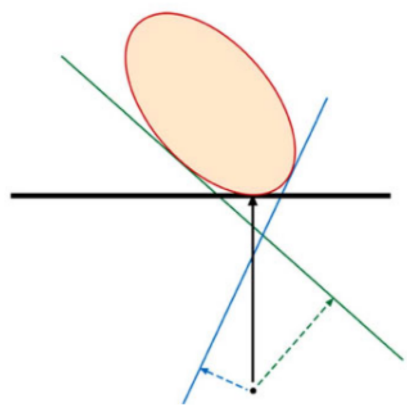

FIGURE 2. Hyperplane choosing

Fig. 2 shows that the lines (hyperplanes) separate the point and the set. To compute the distance, we must choose a point $\boldsymbol{x} \in \mathcal{Y}$ that closest to $\boldsymbol{z} \notin \mathcal{Y}$. The closest point in set $\mathcal{Y}$ is supported by the black hyperplane, which make a maximal separation between the set and the point. Since the hyperplane equation is $\mathcal{H} = \{\boldsymbol{x} | \langle \boldsymbol{a}, \boldsymbol{x} \rangle = \alpha\}$, we must find the vector $\boldsymbol{a}$ such that $\text{dist}(\boldsymbol{z}, \mathcal{H})$ attains the maximal value. Ref. [4] proposed this theorem as the dual of equation (5).

Theorem 5 [4] *Let* $\boldsymbol{\mathcal{A}}$ *denote the set of all points* $\boldsymbol{a}$ *such that hyperplanes* $\boldsymbol{\mathcal{H}_\alpha}$ *and* $\boldsymbol{\mathcal{H}_\beta}$ *separate* $\boldsymbol{\mathcal{Y}}$ *and* $\boldsymbol{\mathcal{Z}}$*. The formula of maximal separation problem is*

$$\begin{aligned} &\text{maximize} \quad \sigma(\boldsymbol{a}) = \beta(\boldsymbol{a}) - \alpha(\boldsymbol{a}) \\ &\text{subject to } \boldsymbol{a} \in \mathcal{A}. \end{aligned} \tag{7}$$

Let $\boldsymbol{a}^*$be solution of this problem, then $\|\boldsymbol{a}^*\|' = 1$, and $\sigma(\boldsymbol{a}^*) = \frac{\beta(\boldsymbol{a}) - \alpha(\boldsymbol{a})}{\|\boldsymbol{a}\|'}$.

Proof. Let $\mathcal{H}_\alpha = \{\boldsymbol{x_1} | \langle \boldsymbol{a}, \boldsymbol{x_1} \rangle = \alpha(\boldsymbol{a})\}$ and $\mathcal{H}_\beta = \{\boldsymbol{x_2} | \langle \boldsymbol{a}, \boldsymbol{x_2} \rangle = \beta(\boldsymbol{a})\}$. It is shown the distance

$$dist(\mathcal{H}_1, \mathcal{H}_2) = \|\boldsymbol{x_1} - \boldsymbol{x_2}\|$$

If $\boldsymbol{a}$ solve (7), then

$$\begin{aligned} \sigma(\boldsymbol{a}) &= \beta(\boldsymbol{a}) - \alpha(\boldsymbol{a}) \\ &= \langle \boldsymbol{a}, \boldsymbol{x_2} \rangle - \langle \boldsymbol{a}, \boldsymbol{x_1} \rangle \\ &= \langle \boldsymbol{a}, \boldsymbol{x_1} - \boldsymbol{x_2} \rangle \\ &= \|\boldsymbol{a}\|' \|\boldsymbol{x_1} - \boldsymbol{x_2}\| \end{aligned}$$

Since $\sigma(\boldsymbol{a}) = \|\boldsymbol{a}\|'\|\boldsymbol{x_1} - \boldsymbol{x_2}\|$, and $dist(\mathcal{H}_1, \mathcal{H}_2) = \|\boldsymbol{x_1} - \boldsymbol{x_2}\|$, then $\sigma(\boldsymbol{a}) = dist(\mathcal{H}_1, \mathcal{H}_2)$ is attained when $\|\boldsymbol{a}\|' = 1$ or $\frac{\beta(\boldsymbol{a})-\alpha(\boldsymbol{a})}{\|\boldsymbol{a}\|'}$.

In practice, it is convenient to write eq. (7) in the form

$$\begin{aligned} &\text{maximize } \ \sigma(\boldsymbol{a}) = \inf_{\boldsymbol{z} \in \mathcal{Z}} \langle \boldsymbol{a}, \boldsymbol{z} \rangle - \sup_{\boldsymbol{y} \in \mathcal{Y}} \langle \boldsymbol{a}, \boldsymbol{y} \rangle \\ &\text{subject to} \|\boldsymbol{a}\|' \leq 1 \end{aligned} \tag{8}$$

The justification for replacing (7) and (8) lies in the definition of $\mathcal{A}$. Recall that $\boldsymbol{a} \in \mathcal{A}$ iff $\sigma(\boldsymbol{a}) \geq 0$. Consequently any solution of (8) solves (7) and vice versa. Ref. [4] gave this theorem for summarizing this result.

Theorem 6 (Dax' Minimum Norm Duality Theorem). *The dual of the least norm problem* (5) *is the maximum separation problem* (8) *and both problems are solvable. Let* $\mathbf{x}^* \in \boldsymbol{\mathcal{X}}$ *solve* (5) *and* let $\mathbf{a}^*$ *solve the* (8), *then*

$$\|\mathbf{a}^*\|' = 1 \textit{ and } \sigma(\mathbf{a}^*) = \|\mathbf{x}^*\| = \text{dist}(\mathcal{Y}, \mathcal{Z})$$

Furthermore, if $\mathbf{x}^* \neq \mathbf{0}$ then $\mathbf{x}^*$ and $-\mathbf{a}^*$ are aligned. That is,

$$\langle -\mathbf{a}^*, \mathbf{x}^* \rangle = \|-\mathbf{a}^*\|'\|\mathbf{x}^*\| = \|\mathbf{x}^*\|$$

SOME EXAMPLES

In this section, it is given some examples to enable us to understand the concept of minimum distance. Let $\mathcal{Y}$ and $\mathcal{Z}$ be two disjoint convex closed sets in Hilbert space. The primal problem to be solved has the form

$$\begin{aligned} &\text{minimize} \|\boldsymbol{y} - \boldsymbol{z}\| \\ &\text{minimize } \boldsymbol{y} \in \mathcal{Y}, \boldsymbol{z} \in \mathcal{Z} \end{aligned} \tag{9}$$

The least distance problems consider some kinds of convex sets, such as normed balls, normed ellipsoids, and polytopes. The study of problems illustrate how the general definition of the dual problem,

$$\begin{aligned} &\text{maximize } \ \sigma(\boldsymbol{a}) = \inf_{\boldsymbol{z} \in \mathcal{Z}} \langle \boldsymbol{a}, \boldsymbol{z} \rangle - \sup_{\boldsymbol{y} \in \mathcal{Y}} \langle \boldsymbol{a}, \boldsymbol{y} \rangle \\ &\text{subject to} \|\boldsymbol{a}\|' \leq 1 \end{aligned} \tag{10}$$

is casted into a specific maximization problem when $\mathcal{Y}$ and $\mathcal{Z}$ are specified.

The examples start by considering two disjoint convex sets, where both $\mathcal{Y}$ and $\mathcal{Z}$ are not singleton. The original problem (9) can be formulated as a best approximation problem to find the shortest distance between two convex sets by computing the distance two nearest points in each set. This gives the problem a second geometric interpretation and enables us to apply the problem (10). Consider for example the distance between two linear varieties, two normed balls, and ellipsoids.

The Distance between Two Linear Varieties

Ref. [12] said that linear variety is obtained by translating linear subspace. The standard form of linear varieties is

$$\mathcal{Y} = \{\tilde{\boldsymbol{y}} + B\boldsymbol{u}\}, \qquad \mathcal{Z} = \{\tilde{\boldsymbol{z}} - C\boldsymbol{v}\}$$

Where $\tilde{\boldsymbol{y}}$ and $\tilde{\boldsymbol{z}}$ are the vector in Hilbert space, which act as translation factors, B and C are the operators, and $B\boldsymbol{u}$ and $C\boldsymbol{v}$ are the linear subspaces. The minimum distance between $\mathcal{Y}$ and $\mathcal{Z}$ has the form

$$\text{minimize } \|\tilde{\boldsymbol{y}} + B\boldsymbol{u} - \tilde{\boldsymbol{z}} + C\boldsymbol{v}\|$$

or, simply,

$$\text{minimize } \|\boldsymbol{b} - A\boldsymbol{x}\| \tag{11}$$

where $\boldsymbol{b} = \boldsymbol{z} - \boldsymbol{y}, A = [B, C]$ and $\boldsymbol{x} = (\boldsymbol{u}, \boldsymbol{v})$ denotes the unknown vector. The problem (11) has a new geometric interpretation, which is to seek the shortest distance between a point $\boldsymbol{b}$ and the subspace

$$\mathcal{S} = \{A\boldsymbol{x}\} = Range(A)$$

To find the distance between $\boldsymbol{b}$ and $\mathcal{S}$, ref. [1] proposed this theorem.

Theorem 7 [1] *Let* $\boldsymbol{b}$ *be an element in a real normed linear space* $\boldsymbol{X}$ *and let* $\mathbf{d}$ *denote its distance from the subspace* $\boldsymbol{\mathcal{S}}$. *Then,*

$$d = \inf_{s \in \mathcal{S}} \|\boldsymbol{b} - \boldsymbol{s}\| = \max_{\substack{\|\boldsymbol{a}^*\| \leq 1 \\ \boldsymbol{a}^* \in \mathcal{S}^\perp}} \langle \boldsymbol{b}, \boldsymbol{a} \rangle \tag{12}$$

Where the maximum on the right side is attained for $\boldsymbol{a}^* \in \mathcal{S}^\perp$, *that is the orthogonal complement of* $\mathcal{S}$.

Proof. This theorem will be proven from the left side to the right side. In problem (12), it is said that $\boldsymbol{s} \in \mathcal{S}$, then $\boldsymbol{s} = A\boldsymbol{x}$. Let $\boldsymbol{w} \in \mathcal{S}^{\perp}$, such that

$$\begin{aligned}\mathcal{S}^{\perp} &= \{\boldsymbol{w} | \langle A\boldsymbol{x}, \boldsymbol{w}\rangle = 0\} \\ &= \{\boldsymbol{w} | \langle \boldsymbol{x}, A^{\dagger}\boldsymbol{w}\rangle = 0\} \\ &= \{\boldsymbol{w} | A^{\dagger}\boldsymbol{w} = \boldsymbol{0}\} = Null(A^{\dagger})\end{aligned}$$

If we apply the formula (8), then $\sigma(\boldsymbol{a}^{*}) \geq 0$. So, $\inf\langle \boldsymbol{b}, \boldsymbol{a}\rangle \geq \sup\langle A\boldsymbol{x}, \boldsymbol{a}\rangle$. Now, let us consider the dual problem. We have two values of $\sup_{\boldsymbol{s} \in \mathcal{S}} \langle \boldsymbol{a}, \boldsymbol{s}\rangle$, that is

$$\sup_{\boldsymbol{s} \in \mathcal{S}} \langle \boldsymbol{a}, \boldsymbol{s}\rangle = \begin{cases} 0, & \boldsymbol{a} \in \mathcal{S}^{\perp} \\ \infty, & otherwise \end{cases}$$

Since $\langle \boldsymbol{b}, \boldsymbol{a}\rangle$ is finite, then $\boldsymbol{a}$ must be belong to the orthogonal complement of $\mathcal{S}$.

Hence by formula (8), the dual of (11) has the form

$$\begin{aligned}&\text{maximize } \langle \boldsymbol{b}, \boldsymbol{a}\rangle \\ &\text{subject to } A^{\dagger}\boldsymbol{a} = \boldsymbol{0}, \|\boldsymbol{a}\|' \leq 1\end{aligned} \tag{13}$$

The Distance between Two Normed Balls

Next, we consider the distance between two normed balls in standard form

$$\mathcal{Y} = \{\boldsymbol{y_c} + \boldsymbol{y} | \|\boldsymbol{y}\| \leq \rho_1\}, \text{ and } \mathcal{Z} = \{\boldsymbol{z_c} - \boldsymbol{z} | \|\boldsymbol{z}\| \leq \rho_2\},$$

where $\boldsymbol{y_c}$ and $\boldsymbol{x_c}$ are the center of the balls, and scalar ρ_1 and ρ_2 are the radii. In this case the least distance problem (9) become

$$\begin{aligned}&\text{minimize} \|\boldsymbol{y_c} - \boldsymbol{z_c} + \boldsymbol{y} + \boldsymbol{z}\| \\ &\text{subject to} \|\boldsymbol{y}\| \leq 1, \|\boldsymbol{z}\| \leq 1\end{aligned} \tag{14}$$

To construct the dual problem, it is necessary to construct two hyperplanes $\alpha(\boldsymbol{a})$ and $\beta(\boldsymbol{a})$ that support $\mathcal{Y}$ and $\mathcal{Z}$ respectively. So,

$$\begin{aligned}\beta(\boldsymbol{a}) &= \inf_{\boldsymbol{z} \in \mathcal{Z}} \langle \boldsymbol{a}, \boldsymbol{z}\rangle = \langle \boldsymbol{a}, \boldsymbol{z_c} - \boldsymbol{z}\rangle \\ &= \langle \boldsymbol{a}, \boldsymbol{z_c}\rangle - \rho_2 \|\boldsymbol{a}\|'\end{aligned}$$

and

$$\begin{aligned}\alpha(\boldsymbol{a}) &= \sup_{\boldsymbol{y} \in \mathcal{Y}} \langle \boldsymbol{a}, \boldsymbol{y}\rangle = \langle \boldsymbol{a}, \boldsymbol{y_c} + \boldsymbol{y}\rangle \\ &= \langle \boldsymbol{a}, \boldsymbol{y_c}\rangle + \rho_1 \|\boldsymbol{a}\|'\end{aligned}$$

So, the dual of (14) has the form

$$\begin{aligned}&\text{maximize } \sigma(\boldsymbol{a}) = -\langle \boldsymbol{a}, \boldsymbol{y_c} - \boldsymbol{z_c}\rangle - \rho_1 \|\boldsymbol{a}\|' - \rho_2 \|\boldsymbol{a}\|' \\ &\text{subject to} \|\boldsymbol{a}\|' \leq 1\end{aligned} \tag{15}$$

To solve problem (15), we use KKT condition $\nabla\sigma + \mu\nabla g = 0$, where σ denotes the objective, μ is the Lagrange multiplier, and g is the constrain. So,

$$\begin{aligned}\nabla\sigma + \mu\nabla g &= 0 \\ -(\boldsymbol{y_c} - \boldsymbol{z_c}) + \frac{\rho_1 + \rho_2 - \mu}{\|\boldsymbol{a}\|} &= 0 \\ \mu &= (\rho_1 + \rho_2) - \|\boldsymbol{z_c} - \boldsymbol{y_c}\|\end{aligned}$$

Since $\mu = (\rho_1 + \rho_2) - \|\boldsymbol{y_c} - \boldsymbol{x_c}\|$, we obtain that $\boldsymbol{a} = \frac{\boldsymbol{z_c} - \boldsymbol{y_c}}{\|\boldsymbol{z_c} - \boldsymbol{y_c}\|}$, and

$$\begin{aligned}\sigma(\boldsymbol{a}) &= -\langle \frac{\boldsymbol{z_c} - \boldsymbol{y_c}}{\|\boldsymbol{z_c} - \boldsymbol{y_c}\|}, \boldsymbol{z_c} - \boldsymbol{y_c}\rangle - \rho_1 \|\boldsymbol{a}\| - \rho_2 \|\boldsymbol{a}\| \\ &= \|\boldsymbol{z_c} - \boldsymbol{y_c}\| - (\rho_1 + \rho_2)\end{aligned}$$

So, the distance of between two circles is equal to $\sigma(\boldsymbol{a}) = \|\boldsymbol{z_c} - \boldsymbol{y_c}\| - (\rho_1 + \rho_2)$.

Now, let us consider if $\mathcal{Z}$ is a singleton, that is $\mathcal{Z} = \{\hat{\boldsymbol{z}}\}$. Then the case (15) become

$$\begin{aligned}&\text{maximize } \sigma(\boldsymbol{a}) = -\langle \boldsymbol{a}, \boldsymbol{y_c} - \hat{\boldsymbol{z}}\rangle - \rho_1 \|\boldsymbol{a}\| \\ &\text{subject to} \|\boldsymbol{a}\|' \leq 1\end{aligned} \tag{16}$$

Similarly, we obtain $\mu = \rho_1 - \|\hat{\boldsymbol{z}} - \boldsymbol{y_c}\|$, $\boldsymbol{a} = \frac{\hat{\boldsymbol{z}} - \boldsymbol{y_c}}{\|\hat{\boldsymbol{z}} - \boldsymbol{y_c}\|}$, and the distance is $\sigma(\boldsymbol{a}) = \|\hat{\boldsymbol{z}} - \boldsymbol{y_c}\| - \rho_1$.

The Distance between Two Normed Ellipsoids

According to ref. [4] and [10], ellipsoids are the sets that denoted by

$$\mathcal{Y} = \{\boldsymbol{y_c} + B\boldsymbol{u} | \|\boldsymbol{u}\| \leq 1\}, \qquad \mathcal{Z} = \{\boldsymbol{z_c} - C\boldsymbol{v} | \|\boldsymbol{v}\| \leq 1\}$$

Where $\boldsymbol{y_c}$ and $\boldsymbol{z_c}$ are the center of ellipsoids, B and C are the operators, and $\|\cdot\|$ denotes the norm. The operator B and C determines how far the ellipsoid extends in every directions from $\boldsymbol{y_c}$, and the length of the semi-axes of $\mathcal{Y}$ and $\mathcal{Z}$ are given by $\sqrt{\kappa_i(B^2)}$ and $\sqrt{\lambda_i(C^2)}$, where $\kappa_i(B^2)$ and $\lambda_i(C^2)$ are the eigenvalues of B^2 and C^2 respectively. If $\mathcal{Y}$ is normed ball, then $B = \rho I$, where I is the identity, and ρ is the radius.

The distance between $\mathcal{Y}$ and $\mathcal{Z}$ is obtained by solving the minimum norm problem

$$\begin{aligned} &\text{minimize } \|\boldsymbol{y_c} - \boldsymbol{z_c} + B\boldsymbol{u} + C\boldsymbol{v}\| \\ &\text{subject to} \|\boldsymbol{u}\| \leq 1 , \|\boldsymbol{v}\| \leq 1 \end{aligned} \tag{17}$$

As before, we can verify that

$$\begin{aligned} \inf_{\boldsymbol{z}\in\mathcal{Z}} \langle \boldsymbol{a}, \boldsymbol{z}\rangle &= \langle \boldsymbol{a}, \boldsymbol{z_c}\rangle + \inf_{\|\boldsymbol{v}\| \leq 1} \langle \boldsymbol{a}, -C\boldsymbol{v}\rangle \\ &= \langle \boldsymbol{a}, \boldsymbol{z_c}\rangle - \|C^\dagger \boldsymbol{a}\|' \end{aligned}$$

and

$$\begin{aligned} \sup_{\boldsymbol{y}\in\mathcal{Y}} \langle \boldsymbol{a}, \boldsymbol{y}\rangle &= \langle \boldsymbol{a}, \boldsymbol{y_c}\rangle + \sup_{\|u\| \leq 1} \langle \boldsymbol{a}, B\boldsymbol{u}\rangle \\ &= \langle \boldsymbol{a}, \boldsymbol{y_c}\rangle + \|B^\dagger \boldsymbol{a}\|' \end{aligned}$$

Where $B^\dagger$ and $C^\dagger$ denote the adjoint of B and C respectively, but if we put matrix as the operator, the adjoint is its transpose. Then, the dual of (17) has the form

$$\begin{aligned} &\text{maximize } \sigma(\boldsymbol{a}) = -\langle \boldsymbol{a}, \boldsymbol{y_c} - \boldsymbol{z_c}\rangle - \|B^\dagger \boldsymbol{a}\|' - \|C^\dagger \boldsymbol{a}\|' \\ &\text{subject to} \|\boldsymbol{a}\|' \leq 1 \end{aligned} \tag{18}$$

Now, consider when $\mathcal{Z}$ turns to singleton, that is $\mathcal{Z} = \{\hat{\boldsymbol{z}}\}$. Then the problem (17) become

$$\begin{aligned} &\text{minimize } \|B\boldsymbol{u} + (\boldsymbol{y_c} - \hat{\boldsymbol{z}})\| \\ &\text{subject to} \|\boldsymbol{u}\| \leq 1 \end{aligned} \tag{19}$$

Let $\boldsymbol{g} = \boldsymbol{y_c} - \hat{\boldsymbol{z}}$, then the dual of (19) has the form

$$\begin{aligned} &\text{maximize } \sigma(\boldsymbol{a}) = -\langle \boldsymbol{g}, \boldsymbol{a}\rangle - \|B^\dagger \boldsymbol{a}\|' \\ &\text{subject to} \|\boldsymbol{a}\|' \leq 1 \end{aligned} \tag{20}$$

Usually in Euclidean space $\mathbb{R}^n$, the operator that used is symmetric and invertible matrix. Below, it is given an example to compute the distance between two ellipsoids that solved by applying KKT Theorem (in ref.[13], p.398). Let $\mathcal{Y}$ and $\mathcal{Z}$ be two ellipsoids in $\mathbb{R}^2$ where $\mathcal{Y} \equiv \frac{{y_1}^2}{4} + {y_2}^2 = 1$ and $\mathcal{Z} \equiv \frac{(z_1-5)^2}{4} + {z_2}^2 = 1$. If we transform to the standard form, the ellipses become

$$\mathcal{Y} = \{\begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} | \left\|\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}\right\| \leq 1\}$$

and

$$\mathcal{Z} = \{\begin{pmatrix} 5 \\ 0 \end{pmatrix} - \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} | \left\|\begin{pmatrix} z_1 \\ z_2 \end{pmatrix}\right\| \leq 1\}$$

By using the dual problem, we compute the distance between $\mathcal{Y}$ and $\mathcal{Z}$ is

$$\begin{aligned} &\text{maximize } \sigma(\boldsymbol{a}) = 5a_1 - 2\sqrt{4a_1^2 + a_2^2} \\ &\text{subject to } a_1^2 + a_2^2 \leq 1 \end{aligned} \tag{21}$$

If we solve the problem (21) by Lagrange multiplier, we obtain that the distance between $\mathcal{Y}$ and $\mathcal{Z}$ is 1.

Next, let us consider the case when $\mathcal{Z}$ be a normed ball, that is $\mathcal{Z} = \{\boldsymbol{z_c} - \rho I\boldsymbol{v} | \|\boldsymbol{v}\| \leq 1\}$. Then, the problem (17) become

$$\begin{aligned} &\text{minimize } \|\boldsymbol{y_c} - \boldsymbol{z_c} + B\boldsymbol{u} + \rho I\boldsymbol{v}\| \\ &\text{subject to} \|\boldsymbol{u}\| \leq 1 , \|\boldsymbol{v}\| \leq 1 \end{aligned} \tag{22}$$

So, the dual of (22) has the form

$$\begin{aligned} &\text{maximize } \sigma(\boldsymbol{a}) = -\langle \boldsymbol{a}, \boldsymbol{y_c} - \boldsymbol{z_c}\rangle - \rho\|\boldsymbol{a}\|' - \|B^\dagger \boldsymbol{a}\|' \\ &\text{subject to} \|\boldsymbol{a}\|' \leq 1 \end{aligned} \tag{23}$$

CONCLUSION

The minimum distance between two convex sets can be measured by determining the closest point among them. One another way is by considering Minimum Norm Duality Theorem, which is by measuring the maximum distance the supporting hyperplane that constructed to separate them. Some references have studied the minimum distance problem in Euclidean space, so they put matrix as the operator. We still apply some rule that stand on Euclidean space to Hilbert space. Further research we will consider the best approximation problem in more specific Hilbert space, such as function space (Lebesgue), matrix space, or sequence space to observe the behavior of best approximation problem in these spaces.

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REFERENCES

1. D.G. Luenberger, *Optimization by Vector Space Methods* (John Wiley & Sons, New York, 1970).
2. H. Edelsbrunner, *Journal of Algorithms*, **6**, pp. 213-224 (1985).
3. W. Briec, *Journal of Optimization Theory and Applications*, **93** (2), pp. 301-319 (1997).
4. A. Dax, *Linear Algebra and its Applications*, pp. 184-213 (2006).
5. F. Deutsch, *Best Approximation in Inner Product Spaces* (Springer-Verlag New York, Inc., New York, 2001).
6. A. Iske, *Approximation Theory and Algorithms for Data Analysis* (Springer Nature Switzerland, Switzerland, 2010).
7. O.L. Mangasarian, *Oper. Res. Lett.*, **24**, pp. 15-23 (1999).
8. J. Hiriart-Urruty and C. Lemarechal, *Fundamentals of Convex Analysis* (Springer-Verlag Berlin, Berlin, 2001).
9. R. Rockafellar, *Convex Analysis Princeton* (Princeton University Press, N.J., 1970).
10. S. Boyd and Vandenberghe, *Convex Optimization* (Cambridge University Press, New York:, 2004).
11. M. Panik, *Fundamental of Convex Analysis: Duality, Separation, Representation, and Resolution* (Springer, Dordrecht, 1993).
12. R.F. Curtain and A.J. Pritchard, *Functional Analysis in Modern Applied Mathematics* (Academic Press, London, 1977).
13. K. Chong and S. Zak, *An Intoduction to Optimization* (John Wiley and Sons, Inc, New Jersey, 2001).

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