LEMBAR
HASIL PENILAIAN SEJAWAT SEBIDANG ATAU *PEER REVIEW*
KARYA ILMIAH : PROSIDING INTERNASIONAL

Judul Prosiding (Artikel)	:	Additional conditions of self-adjoint operator to be applied self-adjoint linear relation on a Hilbert space
Nama/ Jumlah Penulis	:	**Susilo Hariyanto,** R K Sari, Farikhin, Y D Sumanto, Solikhin, A Aziz
Status Pengusul	:	penulis ke-q
Identitas Prosiding	:	a. Nama Prosiding : Journal of Physics: Conference Series
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d. Kelengkapan unsur dan kualitas terbitan/ Prosiding (30%)	8,8	8,975	8,8875
Total = (100%)	**29,3**	**29,8**	**29,55**
Nilai Pengusul = 60% x 29,55 = 17,73			

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NIP. 195809011986032002
Unit Kerja : FSM Undip
Bidang Ilmu: Matematika

Reviewer 2

Nama : Drs. Bayu Surarso, M.Sc Ph.D
NIP. 19631105 198803 1 001
Unit Kerja : FSM Undip
Bidang Ilmu: Matematika

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c. Kecukupan dan kemutahiran data/informasi dan metodologi (30%)	9			8,8
d. Kelengkapan unsur dan kualitas terbitan/prosiding (30%)	9			8,8
Total = (100%)	**30**			**29,3**
Nilai Pengusul = 60% x 29,3 = 17,58				

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1. **<u>Kesesuaian dan kelengkapan unsur isi prosiding:</u>**
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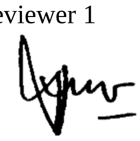

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d. Kelengkapan unsur dan kualitas terbitan/prosiding (30%)	9			8,975
Total = (100%)				**29,8**
Nilai Pengusul = 60% x 29,8 = 17,8				

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Atikel sudah ditulis sesuai panduan penulisan proceeding dan lengkap dari abstrak hingga kesimpulan, namun metodologi penelitian tidak disampaikan secara eksplisit.

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3. Kecukupan dan kemutakhiran data/informasi dan metodologi:
Artikel ini mengangkat topik masalah yang cukup mutakhir. Data literatur dalam daftar pustaka juga beberapa update yakni merupakan publikasi yang baru. Artikel didukung dengan Pustaka sebanyak 14 paper.

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Semarang, Februari 2023
Reviewer 2

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Bidang Ilmu: Matematika

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Additional conditions of self-adjoint operator to be applied self-adjoint linear relation on a Hilbert space

Hariyanto S. ; Sari R.K.; Farikhin; Sumanto Y.D.; Solikhin; Aziz A.
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[a] Department of Mathematics, Faculty of Science and Mathematics, Diponegoro University, Semarang, Indonesia

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Abstract

Let H is a Hilbert space over field real number . An operator T on H is a function from H to H. A Self-adjoint operator is an operator that satisfies T = T∗. Furthermore, a linear relation ϵ on h is the set of pairs of elements w and x with w,x ∈ h. A self-adjoint linear relation is a relation that meets ϵ = ϵ∗. Some properties of a Self-adjoint operator on h is not applicable in self-adjoint linear relation. This paper aims to determine the properties of a self-adjoint linear relation based on linear operators. © Published under licence by IOP Publishing Ltd.

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Preface

It is a very great privilege for Faculty of Mathematics and Natural Science (FMIPA) Universitas Negeri Semarang to host the 5th International Conference on Mathematics, Science, and Education (ICMSE 2018) in Kuta, Bali, Indonesia on 8-9 October 2018. We are honored to have the opportunity to work with Indonesian Chemical Society, Indonesian Physical Society, Indonesian Biology Society, Association of Computer Science Higher Education, Indonesian Mathematical Society, and Association of Indonesian Science Educator in this forum. In 2018, our theme of “Collaborative Research on Science, Mathematics, and Education: Its Application As The Development of Sustainable Resources” celebrates the annual conference to provide a platform to the researchers, experts and practitioners from academia, governments, NGOs, research institutes, and industries to meet and share cutting-edge progress in the field of mathematics, natural science, and science education. Also, this event provides an opportunity to enhance understanding of relationships between knowledge and research in the scope of Mathematics, Biology, Chemistry, Physics, and Science Education.

The committee of ICMSE 2018 would like to express the sincere gratitude to the keynote speakers and all authors of the contributed papers in the conference proceedings. Moreover, would like to thank the expert reviewers for reviewing the manuscripts. We also highly appreciate the assistance offered by many volunteers in the preparation of the conference and the proceedings, and of course, to the sponsors assisting in funding this conference.

The committee selected papers and report findings presented in this forum to be published in **Journal of Physics: Conference Series (Institute of Physics Publisher)** indexed in some databases, including the Conference citation index, Scopus, Inspec, Chemical Abstracts Service, and Astrophysics Data System. We hope that this program will expand the mutual understanding and respect in stimulating research in Mathematics, Science, and Education; share research interest and information, and create a form of collaboration and build a trust relationship. We are delighted to be able to show the world what recent developments in the field of Mathematics, Natural Science, and Science Education through this fruitful program.

Chairperson,

Prof. Dr. St. Budi Waluya

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Papers

Learning differential calculus using self-regulated flipped classroom approach

A Istiandaru[1,*], F Setyawan[1], A S E Hidayat[2] and V Istihapsari[1]

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Abstract. Differential calculus becomes a primary pre-requisite material for every student to start learning calculus. It mainly discusses the concepts and theorems regarding derivative of functions. Many mathematics educators believe that learning differential calculus needs specific conditions and attitude as it cannot be set as rote and procedural learning. This research aims to find out the students' perception towards the implementation of self-regulated flipped classroom approach in their differential calculus class. Thirty-six students participated in the seven meetings of the class and gave their perceptions at the end of every meeting. They were engaged in a various type of learning activities outside the classroom such as setting their own goal, gaining information from many sources, and uploading a video of their presentation, while during the class, they were assessed with an interview confirming their understanding about the topics. The result suggests that the self-regulated flipped classroom approach is promising to maintain the students' right attitude towards differential calculus.

1. Introduction

It is common in the Indonesian mathematics education curriculum that differential calculus is the fundamental subject providing provision for the pre-service teachers to learn calculus and real analysis. The subject is usually taught in the early semester in the teacher training period. The differential calculus subject mainly discusses the concepts and theorems regarding the derivative of functions[1]. Prior to this, the knowledge of equations, functions, limit,and continuity are also important as the pre-requisite material.Further, the students discuss the definition of the derivative, the derivatives in trigonometric functions, the chain rule, the higher order derivatives and the application of derivatives. Since the material of differential calculus set the foundation of analytical thinking, this subject becomes the foundation of logical, critical and creative thinking for the students of mathematics education department[2–4].

The importance of the differential calculus material was not followed by the adequate performance of the students. In the latest two years of teaching differential calculus, mainly using the drill method, we found that the students still confused with the concepts. The students' differential calculus learning result in 2016/2017, for example, shows that only 30% of the students could achieve the score more than 70. We discussed this phenomenon with the other lecturers of the differential calculus and it was confirmed that the similar condition happened in all differential calculus classes. From this score, we

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Nationalism and integrity values in teaching-learning process of mathematics at elementary school of Japan

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Abstract. The values of Nationalism and Integrity are very important to be grown in the young generation. Japan has succeeded in strengthening those values through education in schools. Therefore, it is necessary to do collaborative research between the Research Team of Universitas Negeri Semarang (UNNES) with the partner lecturer from Hiroshima University, that is Prof. Takuya Baba, Ph.D. The problem: How to integrate the values of Nationalism and Integrity in mathematics teaching-learning process at Miyauchi Elementary School of Hiroshima? The research method uses a qualitative approach with the research subjects of Miyauchi Elementary School teachers were selected. The main activity in Japan: observation, interviews, and Focus Group Discussion (FGD) in Miyauchi Elementary School, guided by the partner lecturer. The results of this research: (1) Nationalism and Integrity values has been instilled through families traditionally. (2) Schools apply Nationalism and Integrity values in real context through the learning process, including in the mathematics learning. (3) Courtesy and discipline were also planted in the classroom and has been entrenched among teacher-students. (4) When the student answers the teacher's question, the students immediately stand up and salute by bowing, then respond.

1. Introduction

Schools in Indonesia are currently implementing government programs to cultivate students' character values. There are five main values of the character developed namely: religious, nationalism, autonomy/independent, mutual cooperation, and integrity. Currently schools in Indonesia are starting to implement Strengthening Character Education (PPK). This article, written based on the international collaborative research. Our research team has gone to Hiroshima University with a partner lecturer is Prof. Takuya Baba, Ph.D. Guided by him then the research team conducted next research at Miyauchi Elemetary School of Hiroshima. The study was designed to be implemented for 2 years. Research in the first year focused on the value of nationalism and integrity. This first year of study would like to reveal how the elementary school teachers in Japan integrate the nationalism and integrity values in mathematics learning.

Nationalism is a character value of the way of thinking, behaving, and doing that shows loyalty, caring, and respect which places the interests of the nation and the state above the interests of the self and the group. While Integrity is a character value of the attitude of responsibility as a citizen, actively involved in social life, through the consistency of actions and words based on the truth.

Nationalism and Integrity are very important. Nationalism building and Integrity must be strengthened again in the young generation in Indonesia. And, Japan is a country that teach Nationalism and Integrity values in school specifically in the mathematics learning process from an

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Additional conditions of self-adjoint operator to be applied self-adjoint linear relation on a Hilbert space

by Susilo Hariyanto

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Additional conditions of self-adjoint operator to be applied self-adjoint linear relation on a Hilbert space

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Abstract. Let $\mathfrak{H}$ is a Hilbert space over field real number $\mathbb{R}$. An operator T on $\mathfrak{H}$ is a function from $\mathfrak{H}$ to $\mathfrak{H}$. A Self-adjoint operator is an operator that satisfies $T = T^*$. Furthermore, a linear relation $\mathcal{E}$ on $\mathfrak{H}$ is the set of pairs of elements w and x with $w, x \in \mathfrak{H}$. A self-adjoint linear relation is a relation that meets $\mathcal{E} = \mathcal{E}^*$. Some properties of a Self-adjoint operator on $\mathfrak{H}$ is not applicable in self-adjoint linear relation. This paper aims to determine the properties of a self-adjoint linear relation based on linear operators.

1. Introduction

Arens [1] was the first to introduce linear relation. Recently, there are many studies related to linear relation as reported by Acharya [2], Hassi et al [3], Baskakov and Chernyschov [4], Kascic [5], Gheorghe and Vasilescu [6], and Popovici and Sebestyen [7]. Arens [1] developed self-adjoint linear relation (SALR) based on a self-adjont operator (SAO). Acharya [2] developed some properties of symmetric linear relation which was self-adjoint based on Cayley transformation. Hassi, et al [3] found that a linear relation can be seen as addition of closable operators and singular relation where the closure is a Cartesian product of closed subspace. Hassi, et al [3] found the canonical decomposition properties of a linear relation based linear operator. Kascic [5] found an error in the properties of the linear relation in Arens paper. Furthermore, Kascic rebuilds in a weakened form and applies in a closed linear relation polynomial. Sandovici and Sebestyen [8] found the factorization properties of linear relation in linear space and condition that resulted in the similarity of two linear relations. Miranda [9] found the closure properties of addition and product of linear relations. Langer and Textorious [10] developed SALR by Cayley transformation and Q-function. Sari, et al [12] generalized linear relation as bounded linear operator. The application of linear relation in determining solutions, eigenvalues, and eigenvectors in differential equations can be seen in Baskakov and Chernyschov [4], Gheorghe and Vasilescu [6], and Cross [13].

A linear relation $\mathcal{E}$, or relation for short, on $\mathfrak{H}$ is defined by $\mathcal{E} = \{(w,x), w,x \in \mathfrak{H}\}$ that is a subspace of $\mathfrak{H} \times \mathfrak{H}$. A relation is named self-adjoint if $\mathcal{E} = \mathcal{E}^*$. An adjoint relation $\mathcal{E}^*$ is given by $\mathcal{E}^* = \{(r,s) \in \mathfrak{H}^2 : \langle x,r \rangle = \langle w,s \rangle, \forall (w,x) \in \mathcal{E}\}$. The graph of operator is an example of a linear relation.

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A relation has been widely applied to the problem of determining eigenvalues of a cauchy problem in quantum theory. An Cauchy problems are often found in biology, physics, chemistry, finance, engineering, environment, industry, ecology and others. Given a homogenous abstract Cauchy problem on $\mathfrak{H}$

$$\frac{d}{dt}Mr(t)=Lr(t), t\in\mathbb{R}_+=[0,+\infty) \qquad (1)$$
$$r(0)=r_0$$

where M and L are linear operator on $\mathfrak{H}$. If $N(M)=\{0\}$ then the problem (1) is named non-degenerate. If $N(M)\neq\{0\}$ then the problem (1) is named degenerate. The (1) Cauchy problem can be made in the form

$$\frac{d}{dt}r(t)=M^{-1}Lr(t), t\in\mathbb{R}_+=[0,+\infty) \qquad (2)$$
$$r(0)=r_0$$

and M can be seen as an operator on $\mathfrak{H}$. If $N(M)\neq\{0\}$ then M can not be seen as an eigenvalue operator, so that the problem (2) take form

$$\frac{d}{dt}r(t)\in M^{-1}Lr(t), t\in\mathbb{R}_+=[0,+\infty) \qquad (3)$$
$$r(0)=r_0$$

where $M^{-1}L$ is a relation on $\mathfrak{H}$. The properties of SAO are not all applicable in SALR. This paper aims to give an additional condition for some properties of SAO that can not be applied in SALR to apply in SALR and determine some properties of SAO that can be applied in SALR. This paper consists of two sections as follows. We give concept and notation of a relation used on $\mathfrak{H}$ in section 2. We give some properties of SALR on $\mathfrak{H}$ based on SAO in section 3

2. Preliminaries

In this section given some properties of SAO on $\mathfrak{H}$ can be seen in [16] and notations of relation on $\mathfrak{H}$ as can be seen in [1,2,3,11].

2.1. Self-Adjoint Operator

A Hilbert space $\mathfrak{H}$ in this paper is assumed over the field $\mathbb{K}$ ($\mathbb{R}$ or $\mathbb{C}$). An operator on $\mathfrak{H}$ is function from $\mathfrak{H}$ to $\mathfrak{H}$. An operator on $\mathfrak{H}$ is named linear if $L(w+x)=L(w)+L(x)$ and $L(\alpha w)=\alpha L(w)$ for all $w,x\in D(L)$ and scalars $\alpha\in\mathbb{K}$. If there exists a real number C such that $\|Lw\|\le C\|w\|$ then an operator on $\mathfrak{H}$ is named bounded if there is L on $\mathfrak{H}$ is named self-adjoint if $L^*=L$.

The following lemma and theorems of SAO, together with their proof can be found [16].

Theorem 1 Let L and S are SAO. Then LS is self-adjoint if and only if the operators commute,

$$LS=SL. \qquad (4)$$

Proposition 2 Let L be a bounded linear operator on $\mathfrak{H}$. Then $L=0$ if and only if $\langle Lw,w\rangle=0, \forall w\in\mathfrak{H}$.

Theorem 3 Let L be a bounded linear operator on $\mathfrak{H}$. Then L^*L is positive.

Theorem 4 Let L and S be positive SAO with $0\le L^2\le S^2$. Then $0\le L\le S$ is positive.

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2.2. Linear Relation

A linear relation, or relation for short, on $\mathfrak{H}$ is denoted $\mathcal{E}=\{(w,x),w,x\in\mathfrak{H}\}$, that is a subspace of $\mathfrak{H}\oplus\mathfrak{H}$. The class of all linear relation on $\mathfrak{H}$ will be denoted by $LR(\mathfrak{H})$.

The domain of $\mathcal{E}$ is defined by $D(\mathcal{E})=\{w\in\mathfrak{H},(w,x)\in\mathcal{E}\}$. The range of $\mathcal{E}$ is defined by $R(\mathcal{E})=\{x\in\mathfrak{H},(w,x)\in\mathcal{E}\}$. The kernel of $\mathcal{E}$ is defined by $N(\mathcal{E})=\{w\in\mathfrak{H},(w,0)\in\mathcal{E}\}$. The multivalued part of $\mathcal{E}$ is defined by $M(\mathcal{E})=\{x\in\mathfrak{H},(0,x)\in\mathcal{E}\}$. The inverse of $\mathcal{E}$ is a relation of $\mathcal{E}^{-1}$ denoted by $\mathcal{E}^{-1}=\{(x,w),(w,x)\in\mathcal{E}\}$. Furthermore, the duality of $\mathcal{E}$ and its inverse $\mathcal{E}^{-1}$ is given by

$$D(\mathcal{E}^{-1})=R(\mathcal{E}),R(\mathcal{E}^{-1})=D(\mathcal{E}),N(\mathcal{E}^{-1})=M(\mathcal{E}),M(\mathcal{E}^{-1})=N(\mathcal{E}). \quad (5)$$

An adjoint relation $\mathcal{E}^*$ is a closed relation given by $\mathcal{E}^*=\{(r,s)\in\mathfrak{H}^2:\langle x,r\rangle=\langle w,s\rangle,\forall(w,x)\in\mathcal{E}\}$. Let $\mathcal{E},\mathcal{J}\in LR(\mathfrak{H})$, then the sum $\mathcal{E}+\mathcal{J}$ is a relation on $\mathfrak{H}$ defined by

$$\mathcal{E}+\mathcal{J}=\{(w,x+l):(w,x)\in\mathcal{E},(w,l)\in\mathcal{J}\}. \quad (6)$$

The product (composition) $\mathcal{J}\mathcal{E}$ is a relation on $\mathfrak{H}$ defined by

$$\mathcal{J}\mathcal{E}=\{(w,l):\exists x\in\mathfrak{H},(w,x)\in\mathcal{E},(x,l)\in\mathcal{J}\}. \quad (7)$$

The relation $z\mathcal{E}$ for $z\in\mathbb{K}$ is defined by $z\mathcal{E}=\{(w,zx):(w,x)\in\mathcal{E}\}$ and the relation $z-\mathcal{E}$ is defined by $z-\mathcal{E}=\{(w,zw-x):(w,x)\in\mathcal{E}\}$.

A relation $\mathcal{E}$ is named symmetric if $\mathcal{E}\subset\mathcal{E}^*$. A relation $\mathcal{E}$ is named self-adjoint if $\mathcal{E}=\mathcal{E}^*$. If $\langle w,r\rangle=\langle x,s\rangle,\forall(w,x),(r,s)\in\mathcal{E}$, then a relation $\mathcal{E}$ is an isometry. Furthermore, If relation $\mathcal{E}$ is an isometry and $D(\mathcal{E})=R(\mathcal{E})=\mathfrak{H}$ then $\mathcal{A}$ is unitary.

A relation $\mathcal{E}$ is a graph of an operator if only if $M(\mathcal{E})=\{0\}$. If $R(\mathcal{E})=\mathfrak{H}$, then a relation $\mathcal{E}$ is a surjective. If $N(\mathcal{E})=\{0\}$, then a relation $\mathcal{E}$ is an injective. If $D(\mathcal{E})=\mathfrak{H}$ and $\|\mathcal{E}\|<\infty$ then $\mathcal{E}$ is bounded.

3. Result and Discussion

In this section we give an additional condition for some properties of SAO that can not applied in SALR to apply in SALR and determine some properties of SAO that can be applied in SALR. Theorem 1 and Proposition 2 are not applicable in SALR. We give the following example Theorem 1 is not applicable in SALR.

Consider the example shows that the properties $\mathcal{E}\mathcal{J}=\mathcal{J}\mathcal{E}$ is not applicable in relations.

Example 5 Given SALR $\mathcal{E}=\{(1,3),(2,4),(3,5),(4,6),(5,7)\}$ and $\mathcal{J}=\{(1,1),(2,4),(3,9),(4,16),(5,25)\}$. Clearly, $\mathcal{E}\mathcal{J}=\{(1,3),(2,6)\}$ and $\mathcal{J}\mathcal{E}=\{(1,9),(2,16),(3,25)\}$. Clearly, $\mathcal{E}\mathcal{J}\neq\mathcal{J}\mathcal{E}$. Given SALR $\mathcal{A},\mathcal{B}\in LR(\mathfrak{H})$, where $\mathcal{A}=\left\{\left(w,\frac{1}{2}w\right),w\in\mathbb{R}\right\}$ and $\mathfrak{B}=\left\{(\frac{1}{2}w,w^2),w\in\mathbb{R}\right\}$. Furthermore, $\mathfrak{B}\mathcal{A}=\{(w,w^2),w\in\mathbb{R}\}$ but $\mathcal{A}\mathfrak{B}=\left\{\left(\frac{1}{2}w_B,\frac{1}{2}w_A\right):w_A=w_B{}^2\right\}$. Clearly, $\mathcal{A}\mathfrak{B}\neq\mathfrak{B}\mathcal{A}$. Given SALR $\mathcal{J},\mathcal{K}\in LR(\mathfrak{H})$, where $\mathcal{J}=\{(1,0),(2,1),(3,2),(4,3)\}$ and $\mathcal{J}=\mathcal{K}$. Furthermore, $\mathcal{J}\mathcal{K}=\{(2,0),(3,1),(4,2)\}$ but $\mathcal{K}\mathcal{J}=\{\{(2,0),(3,1),(4,2)\}\}$. Clearly, $\mathcal{J}\mathcal{K}=\mathcal{K}\mathcal{J}$. Consequently, Theorem 1 can not be applied in SALR.

Based on example 5, we give additional condition in Theorem 1 to apply in SALR. We give the following theorem.

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Theorem 6 Let relation $\mathcal{E}$ and $\mathcal{J}$ on $\mathfrak{H}$ are a self-adjoint. The product of two bounded SALR $\mathcal{E}$ and $\mathcal{J}$ on $\mathfrak{H}$ is self-adjoint, $\mathcal{J} \subset \mathcal{E}, D(\mathcal{E}) = D(\mathcal{J})$, and $M(\mathcal{E}) = M(\mathcal{J})$, if and only if $\mathcal{EJ} = \mathcal{JE}$.

Proof. $(\Rightarrow)$ Let $(w,x) \in \mathcal{E}$, so that $w \in D(\mathcal{E}) = D(\mathcal{J})$. Then there exists $x_1 \in R(\mathcal{J})$ such that $(w,x_1) \in \mathcal{J} \subset \mathcal{E}$. Furthermore, $(0, x_1 - x) = (w,x_1) - (w,x) \in \mathcal{E}$, so that $x_1 - x \in M(\mathcal{E}) = M(\mathcal{J})$. Clearly, we get $(0, x_1 - x) \in \mathcal{J}$. Therefore, we get $(w,x) = (w,x_1) - (0,x_1 - x) \in \mathcal{J}$. Consequently, since $\mathcal{E} = \mathcal{J}$, obtained $\mathcal{EJ} = \mathcal{JE}$. Thus, the product of two bounded SALR $\mathcal{E}$ and $\mathcal{J}$ on $\mathfrak{H}$ is self-adjoint, $\mathcal{J} \subset \mathcal{E}, D(\mathcal{E}) = D(\mathcal{J})$, and $M(\mathcal{E}) = M(\mathcal{J})$, then $\mathcal{EJ} = \mathcal{JE}$.

$(\Leftarrow)$ Let relation $\mathcal{E}$ and $\mathcal{J}$ on $\mathfrak{H}$ are a self-adjoint. Clearly, we have $\mathcal{EJ} = \{(k,x) : \exists w = l, \forall (w,x) \in \mathcal{E}, (k,l) \in \mathcal{J}\}$ and $\mathcal{JA} = \{(w,l) : \exists k = x, \forall (w,x) \in \mathcal{E}, (k,l) \in \mathcal{J}\}$. Since $\mathcal{EJ} = \mathcal{JE}$, we have $k = w$ and $l = x$. Consequently, $\mathcal{J} \subset \mathcal{E}$, $D(\mathcal{E}) = D(\mathcal{J})$, and $M(\mathcal{E}) = M(\mathcal{J})$. Furthermore, $(k,x) \in \mathcal{EJ}$ so that $(k,x) \in (\mathcal{EJ})^*$. Since $\mathcal{EJ} = (\mathcal{EJ})^*$, we have a relation $\mathcal{EJ}$ is self-adjoint. Furthermore, $(w,l) \in \mathcal{JE}$ so that $(w,l) \in (\mathcal{JE})^*$. Since $\mathcal{JE} = (\mathcal{JE})^*$, we have a relation $\mathcal{JE}$ is self-adjoint. Thus, if $\mathcal{EJ} = \mathcal{JE}$ then the product of two bounded SALR $\mathcal{E}$ and $\mathcal{J}$ on $\mathfrak{H}$ is self-adjoint, $\mathcal{J} \subset \mathcal{E}, D(\mathcal{E}) = D(\mathcal{J})$, and $M(\mathcal{E}) = M(\mathcal{J})$.

Thus, the product of two bounded SALR $\mathcal{E}$ and $\mathcal{J}$ on $\mathfrak{H}$ is self-adjoint, $\mathcal{J} \subset \mathcal{E}, D(\mathcal{E}) = D(\mathcal{J})$, and $M(\mathcal{E}) = M(\mathcal{J})$, if and only if $\mathcal{EJ} = \mathcal{JE}$.

We give the following example that Proposition 2 is not applicable in SALR.

Example 7:

Given relation $\mathcal{E}_1 = \{(w,0) : w \in \mathbb{R}\}$. A relation $\mathcal{E}$ on $\mathfrak{H}$ is self-adjoint, that is $\mathcal{E}_1 = \mathcal{E}^*{}_1$. Furthermore, $0 \in \mathcal{E}_1 w$ then $\langle \mathcal{E}_1 w, w \rangle = 0, \forall w \in \mathbb{R}$. Otherwise, given relation $\mathcal{E}_2 = \{(4,-2),(4,2),(0,0),(1,-1),(1,1)\}$. Furthermore, we get $\langle \mathcal{E}_2 w, w \rangle = 4.1 + 4(-2) + 0.0 + 1.(-1) + 1.1 = 0$. Clearly, there exists $\mathcal{E}_2 w \neq 0$ so that $\langle \mathcal{E}_2 w, w \rangle = 0$.

Based on example 7, we give condition in Proposition 2 to be applied in SALR. We give the following theorem.

Proposition 8 Let SALR $\mathcal{E}$ is a bounded on $\mathfrak{H}$. If $0 \in \mathcal{E}w$ then $\langle \mathcal{E}w, w \rangle = 0, \forall w \in \mathfrak{H}$.

Proof. A relation $\mathcal{E}$ is self-adjoint that is $\mathcal{E} = \mathcal{E}^*$. Furthermore, if $0 \in \mathcal{E}w$ then $\langle \mathcal{E}w, w \rangle = \mathcal{E}w_1.w_1 + \mathcal{E}w_2.w_1 + \ldots + \mathcal{E}w_n.w_n = 0$. Thus, if $0 \in \mathcal{E}w$ then $\langle \mathcal{E}w, w \rangle = 0, \forall w \in \mathfrak{H}$.

Let a relation $\mathcal{E}$ is self-adjoint. We give that $\mathcal{E}$ is positive denoted $\mathcal{E} \geq 0$ if $\langle x, w \rangle \geq 0$ for each $w, x \in \mathfrak{H}$. We give the following properties of SALR.

Theorem 9 Let relation $\mathcal{E}$ on $\mathfrak{H}$ is self-adjoint. The following statements are true.

1. $\|\mathcal{E}\| \leq 1$ if and only if $-I \leq \|\mathcal{E}\| \leq I$.
2. $O \leq \|\mathcal{E}\| \leq I \Leftrightarrow O \leq \|\mathcal{E}\|$ and $\|\mathcal{E}\| \leq 1 \Leftrightarrow \mathcal{E}^2 \leq \mathcal{E}$.

Proof. (1)Let relation $\mathcal{E}$ is self-adjoint then $\langle x, w \rangle \in \mathbb{R}$ get

$$\begin{aligned} \langle x - w, w \rangle &= \langle x, w \rangle - \|w\|^2 \\ &\leq |\langle x, w \rangle| - \|w\|^2 \\ &\leq (\|x\| - \|w\|)\|w\| \end{aligned} \tag{8}$$

If $\|x\| \le \|w\|$ for each $x, w \in \mathfrak{H}$ then $(\mathcal{E} - I) \le O$ which is $\|\mathcal{E}\| \le 1$ implies $-I \le \|\mathcal{E}\| \le I$. Otherwise, If $(\mathcal{E} - I) \le O$ then $\langle x, w \rangle \le \|w\|^2$. Furthermore, we have $\sup_{\|w\|=1} |\langle x, w \rangle| \le 1$. Since $\mathcal{E}$ is self-adjoint then $\|\mathcal{E}\| = \sup_{\|w\|=1} |\langle x, w \rangle|$. Furthermore, $-I \le \|\mathcal{E}\| \le I$ implies $\|\mathcal{E}\| \le 1$.

(2) Let $O \le \|\mathcal{E}\| \le I$ if and only if $O \le \|\mathcal{E}\|$ and $\|\mathcal{E}\| \le 1$. If $O \le \|\mathcal{E}\|$ and $\|\mathcal{E}\| \le 1$ then $\mathcal{E}$ is a nonnegative contractive. Clearly, $\langle x, x \rangle = \|x\|^2 \le \langle x, w \rangle$ for some $w, x \in \mathfrak{H}$, so that $\mathcal{E}^2 \le \mathcal{E}$. Conversely, if relation $\mathcal{E}$ on $\mathfrak{H}$ is self-adjoin and $\mathcal{E}^2 \le \mathcal{E}$ then $\|x\|^2 = \langle x, x \rangle \le \langle x, w \rangle \le \|x\| \|w\|$ and hence $\|x\| \le \|w\|$ that is $\|\mathcal{E}\| \le 1$. Thus $O \le \|\mathcal{E}\|$ and $\|\mathcal{E}\| \le 1$ if and only if $\mathcal{E}^2 \le \mathcal{E}$.

Theorem 10 Let $\mathcal{J}$ and $\mathcal{E}$ be positive SALR on $\mathfrak{H}$ with $0 \le \mathcal{E}^2 \le \mathcal{J}^2$. Then $0 \le \mathcal{E} \le \mathcal{J}$ is a positive.

Proof. Let $\mathcal{J} = \{(w, x) : w, x \in \mathfrak{H}\}$ and $\mathcal{E} = \{(r, s) : r, s \in \mathfrak{H}\}$ be positive SALR with $0 \le \mathcal{A}^2 \le \mathfrak{B}^2$. Clearly, we get

$$0 \le \mathcal{E}^2 \le \mathcal{J}^2 \Leftrightarrow 0 \le \langle s, r \rangle^2 \le \langle x, w \rangle^2 . \tag{9}$$

$$\Leftrightarrow 0 \le \langle s, r \rangle \le \langle x, w \rangle$$

$$\Leftrightarrow 0 \le \mathcal{E} \le \mathcal{J} .$$

Furthermore, if $\langle s, r \rangle^2$ is a positive then $\langle s, r \rangle$ is also positive, if $\langle s, r \rangle^2$ is a positive then $\langle s, r \rangle$ is also positive. Thus, $0 \le \mathcal{E} \le \mathcal{J}$ is a positive.

Theorem 11 Let a relation $\mathcal{E}$ is self-adjoint on $\mathfrak{H}$. Then $\mathcal{E}^* \mathcal{E}$ is positive.

Proof. Given $(w, x) \in \mathcal{E}$ and $(k, l) \in \mathcal{E}^*$. A relation $\mathcal{E}$ on $\mathfrak{H}$ is self-adjoint, that is $\mathcal{E} = \mathcal{E}^*$. Clearly, $\mathcal{E}^* \mathcal{E} = \{(w, l) : x = k, (w, x) \in \mathcal{E}, (k, l) \in \mathcal{E}^*\}$ implies $\langle (\mathcal{E}^* \mathcal{E}) w, w \rangle = \langle \mathcal{E}^* w, \mathcal{E}^* w \rangle = \langle l, l \rangle = \|l\|^2 \ge 0$. Thus, $\langle (\mathcal{E}^* \mathcal{E}) w, w \rangle \ge 0$ then $\mathcal{E}^* \mathcal{E}$ is positive. Thus, let a relation $\mathcal{E}$ is self-adjoint on $\mathfrak{H}$, then $\mathcal{E}^* \mathcal{E}$ is positive.

Conclusion

There are properties of linear operators that cannot be applied in SALR. Additional requirements are needed to apply to relations, so that we get the properties of relations. Let relations $\mathcal{E}$ and $\mathcal{J}$ on $\mathfrak{H}$ are a self-adjoint. The product of two bounded self-adjoint relations $\mathcal{E}$ and $\mathcal{J}$ on $\mathfrak{H}$ is self-adjoint, $\mathcal{J} \subset \mathcal{E}, D(\mathcal{E}) = D(\mathcal{J})$, and $M(\mathcal{E}) = M(\mathcal{J})$, if and only if $\mathcal{E}\mathcal{J} = \mathcal{J}\mathcal{E}$. Let SALR $\mathcal{E}$ is a bounded on $\mathfrak{H}$. If $0 \in \mathcal{E}w$ then $\langle \mathcal{E}w, w \rangle = 0, \forall w \in \mathfrak{H}$. There are properties of linear operators that can be applied in linear relations, including Theorem 9, Theorem 10, and Theorem 11.

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