LQR CONTROL FOR A SMALL SCALE HELICOPTER IN HOVER FLIGHT CONDITION

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Abstract

Small scale helicopters have been used as unmanned aerial vehicle (UAV) because they have agility and maneuverability that make them as an ideal option for various missions ranging from weather research, agriculture, aerial surveillance to power line inspection. Small scale helicopter posses a higher bandwidth of dynamics and a greater sensitivity to control inputs which make them more difficult to control. This paper deals with the control system design using Linear Quadratic Regulator (LQR) for an autonomous small scale helicopter in hover flight condition. A nonlinear dynamics model of the small scale helicopter is derived from the Euler-Newton equations of motion. Linear model at hover flight condition is numerically extracted using MATLAB/Simulink. Linear control system is then designed for the small scale helicopter with the following predefined hover position and external disturbances such as longitudinal doublet input and gust. The Virtual Reality model in Matlab/Simulink is used to show the 3 D view of the simulation results.

Kata kunci: *LQR, hover, small scale helicopter,*

1. INTRODUCTION

Small scale helicopter posses higher bandwidth, high order nonlinear, hybrid modes, non-holonomic, under-actuation, multi-input-multi-output, and non-minimum phase which make it more challenging to control. The ability of small scale helicopter to fly autonomously is the key factor. This report uses LQR control technique to control small scale helicopter in hover flight condition using X-cell 60 SE helicopter model as shown in figure 1. The LQR is chosen as the design standard due to its proven robustness for autonomous control of aerobatic maneuvers previously demonstrated by team at MIT [3]. X cell 60 SE helicopter model is characterized by hinge-less rotor with a diameter of 0.775 m and mass of 8 kg. The X-Cell blades both for main and tail rotors use symmetric airfoils.

Figure 1: Instrumented X-Cell 60 helicopter [2]

2. NONLINEAR MODEL OF SMALL SCALE HELICOPTER

The X cell 60 SE model helicopter parameters that are used for simulation in this paper and technical explanation of these parameters can be seen in references [2, 3]. The helicopter moving with 6 DOFs requires six nonlinear differential equations to represent its translational motions and angular motions in respect to 3 axes of references on the helicopter body coordinate. In the equations u, v, w are the linear velocities of the fuselage while p, q, r are the angular rates of body coordinate system. In this X-cell 60 SE helicopter, the value of I_{xz} is negligible [2]

$$\dot{u} = vr - wq - g\sin\theta + X/m \quad (1)$$

$$\dot{v} = wp - ur - g\cos\theta\sin\phi + Y/m \quad (2)$$

$$\dot{w} = uq - vp - g\cos\theta\cos\phi + Z/m \quad (3)$$

$$\dot{p} = qr(I_{yy} - I_{zz})/I_{xx} + L/I_{xx} \quad (4)$$

$$\dot{q} = rp(I_{zz} - I_{xx})/I_{yy} + M/I_{yy} \quad (5)$$

$$\dot{r} = pq(I_{zz} - I_{yy})/I_{zz} + N/I_{zz} \quad (6)$$

External forces and moments have an effect on helicopter components: main rotor, tail rotor, fuselage, horizontal fin, vertical fin, and gravitational force. For further development of these forces and moment is found in [2,3]

$$\begin{aligned} X &= X_{mr} + X_{fus} \\ Y &= Y_{mr} + Y_{fus} + Y_{tr} + Y_{vf} \\ Z &= Z_{mr} + Z_{fus} + Z_{hf} \\ L &= L_{mr} + L_{vf} + L_{tr} \\ M &= M_{mr} + M_{hf} \\ N &= -Q_e + L_{vf} + N_{tr} \end{aligned} \quad (7)$$

Transformation of speed and orientation into a fixed coordinate on earth is performed in [1] as follows.

$$\begin{aligned} V_x &= \frac{dx}{dt} = u\cos\theta\cos\psi + v(\sin\phi\sin\theta\cos\psi - \cos\varphi\sin\psi) + w(\cos\varphi\sin\theta\cos\psi + \sin\phi\sin\psi) \\ V_y &= \frac{dy}{dt} = u\cos\theta\sin\psi + v(\sin\phi\sin\theta\sin\psi + \cos\varphi\cos\psi) + w(\cos\varphi\sin\theta\sin\psi - \sin\phi\cos\psi) \\ V_z &= \frac{dz}{dt} = -u\sin\theta + v\sin\phi\cos\theta + w\cos\varphi\cos\theta \end{aligned} \quad (8)$$

$$\begin{aligned} \dot{\phi} &= p + (q\sin\phi + r\cos\phi)\tan\theta \\ \dot{\theta} &= q\cos\phi + r\sin\phi \\ \dot{\psi} &= (q\sin\phi + r\cos\phi)\sec\theta \end{aligned} \quad (9)$$

For external forces computation that come from main rotor, tail rotor, fuselage, vertical fin, and horizontal fin more detail at reference [2] and [3].

3. LINEAR MODEL IN HOVER FLIGHT CONDITION

Small scale helicopter in hover flight condition can be characterized as followed:

$$\begin{array}{lllll} \dot{u} = 0 & \dot{p} = 0 & u = u_o = 0 & p = p_o = 0 & \phi = \phi_o \\ \dot{v} = 0 & \dot{q} = 0 & v = v_o = 0 & q = q_o = 0 & \theta = \theta_o \\ \dot{w} = 0 & \dot{r} = 0 & w = w_o = 0 & r = r_o = 0 & \psi = \psi_o \end{array} \quad (10)$$

Linear model of nonlinear model is numerically extracted using MATLAB/Simulink at operating point in hover. For detail developing of operating point in hover flight condition can be seen at reference [2]. The results of A and B matrices are:

A matrices

-0.0115	0.0002	0	-9.8100	-21.2001	0	0	0	0	0	0	0	0	0
0.0337	0.0123	0	0.0142	0.0302	0.0337	0	0	-0.7609	0.1587	0	0	0	0
-0.0002	-0.0005	0.0001	0	278.1601	-0.0003	0	0	0	0	0	0	0	0
0	0	0.9970	0	0	0	0	-0.0776	0	0	0	0	0	0
0.0059	0	-1.0000	0	-8.3500	0	0	0	0	0	0	0	0	0
-0.0080	-0.0146	-0.0130	0.0011	0	0.0614	0.0071	-0.0791	9.7804	21.1995	0	0	0	0
-0.0307	-0.0583	-0.0470	0	0	0.3124	0.0257	-0.2867	0	524.0414	0	0	0	0
0.2047	0.3785	0.3445	0	0	-2.3087	-0.1884	2.1009	0	0	0	0	0	0
0	0	-0.0001	0	0	0	1.0000	-0.0014	0	0	0	0	0	0
0	0	0	0	0	0.0059	-1.0000	0	0	-8.3500	0	0	0	0
0	0	0.0776	0	0	0	0	0.9970	0	0	0	0	0	0
1.0000	-0.0014	0	0	0	-0.0001	0	0	0	0	0	0	0	0
0	-0.0776	0	0	0	0.9970	0	0	0	0	0	0	0	0
0.0014	0.9970	0	0	0	0.0776	0	0	0	0	0	0	0	0

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B matrices

$$
\begin{bmatrix}
-0.2817 & 0 & 0 & 0 \\
-197.37 & 0 & 0 & 0 \\
1.591 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
-0.0003 & 35.07 & 0 & 0 \\
1.47789 & 0 & -154.705 & 0 \\
15.7383 & 0 & -560.7 & 0 \\
0.00222 & 0 & 4108.228 & 0 \\
0 & 0 & 0 & 0 \\
-0.0003 & 0 & 0 & 35.07 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
$$

4. LQR CONTROL

The state space equation for a linear time invariant system.

$$
\begin{aligned}
\dot{x}(t) &= Ax(t) + Bu(t) \\
y(t) &= Cx(t) + Du(t)
\end{aligned} \tag{11}
$$

can be evaluated using the cost function

$$
J(x,t) = \int_0^\infty (x^T Q x + u^T R u)\, d\tau \tag{12}
$$

to find its optimal control using LQR method, where $Q=Q^T \geq 0$ is symmetric and positif semi definite matrix, $R=R^T > 0$ is also is symmetric and positif semi definite matrix. Assuming all states area available, C is then set to an identity matrix. The LQR method is solved by the Riccati Equation

$$
\dot{S}(t) = -(S(t)A + A^T S(t) + Q - S(t)BR^{-1}B^T S(t)) \tag{13}
$$

The value of k matrix for the LQR control can be calculated using

$$
u^* = -R^{-1}B^T S(t)x \tag{14}
$$

The state x and input u below are used,

$x = [\, u\ w\ q\ \theta\ a_{1s}\ v\ p\ r\ \varphi\ b_{1s}\ x\ y\ z\ \psi \,]^T$

$u = [\delta_{col}\delta_{long}\delta_{ped}\delta_{lat}]^T$

Table 1: Closed loop eigen value, damping and frequency at hover

Eigenvalue	Damping	Freq. (rad/s)
-1.53e+002	1.00e+000	1.53e+002
-4.17e+001 + 3.70e+001i	7.48e-001	5.57e+001
-4.17e+001 - 3.70e+001i	7.48e-001	5.57e+001
-5.44e+001	1.00e+000	5.57e+001
-2.59e+001 + 2.31e+001i	7.46e-001	3.48e+001
-2.59e+001 - 2.31e+001i	7.46e-001	3.48e+001
-3.61e+001	1.00e+000	3.61e+001
-2.02e+000 + 2.77e+000i	5.90e-001	3.43e+000
-2.02e+000 - 2.77e+000i	5.90e-001	3.43e+000
-2.01e+000 + 2.72e+000i	5.90e-001	3.38e+000
-2.01e+000 - 2.72e+000i	5.90e-001	3.38e+000
-2.66e+000	1.00e+000	2.66e+000
-2.65e+000	1.00e+000	2.65e+000
-3.17e+000	1.00e+000	3.17e+000

After several trials and errors, the matrix Q and R are chosen
Q=diag([0.1 0.1 0.1 0.1 1 0.1 0.1 0.00000001 0.1 0.1 0.1 1 1 1]);
R=diag([1 1 1 1]);
Klqr=lqr(A,B,Q,R);

Table 1 shows that the highest eigenvalue is -153 at frequency 153 rad/sec, and the lowest eigenvalue is -2.01- 2.72i at frequency 3.17 rad/sec. The minimum time time constan is about 0.0065 second and the maximum time constan is about 0.377 second.

The result of Klqr is applied at nonlinear model to maintain hover position. The simulation results are shown in figures 2 through 7.

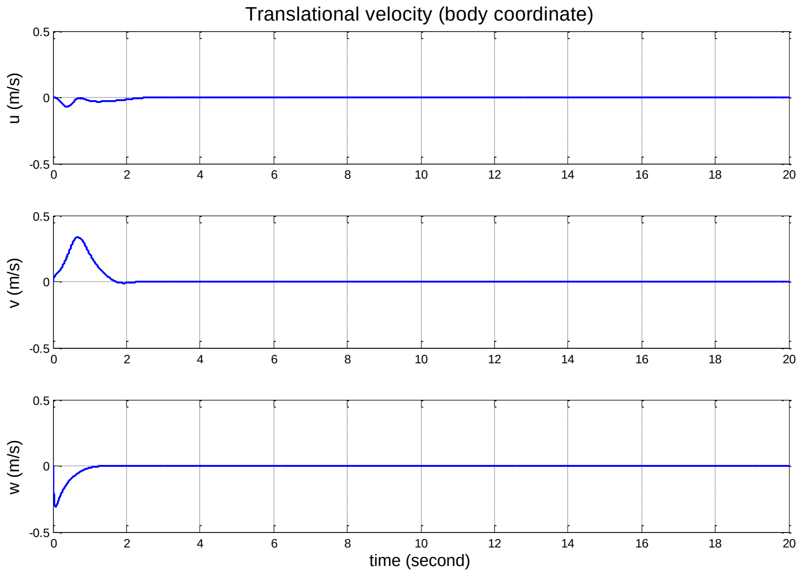

Figure 2: Translational velocities during full state feedback controlled hover

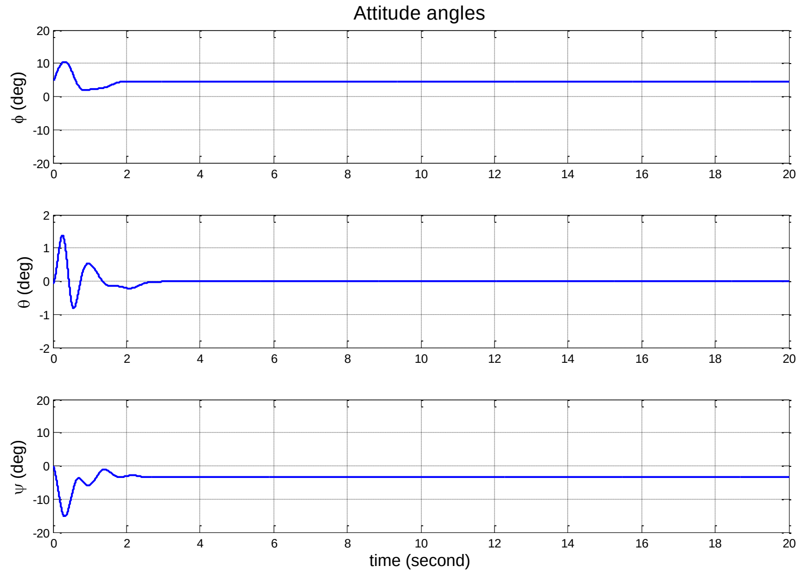

Figure 3: Altitude angles during full state feedback controlled hover

The closed loop response as shown in figures 2, 3, and 4 reveals that translational velocity, alltitude angle and angular rate responses maintain around operating points in hover flight condition so the closed loop is stable.

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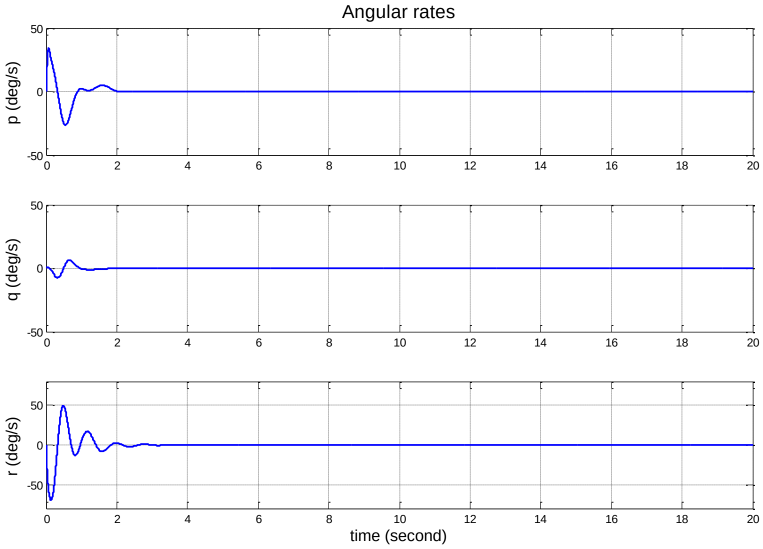

Figure 4: Angular rates during full state feedback controlled hover

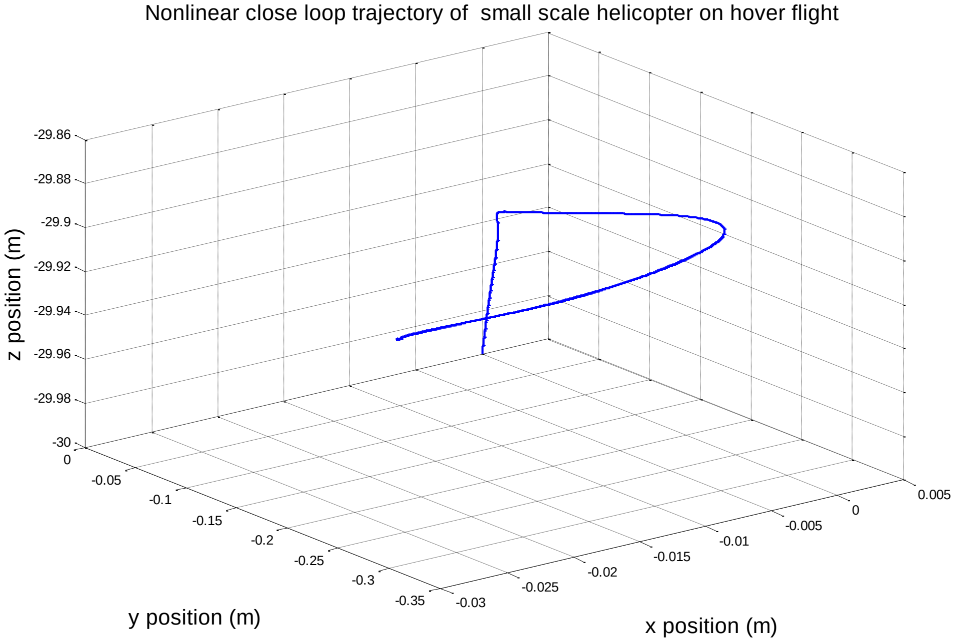

Figure 5: small scale helicopter trajectory during full state feedback controlled hover

It can be seen in Figures 5 that the closed loop system is stable. However, some drift occur on the translational X-Y-Z position.

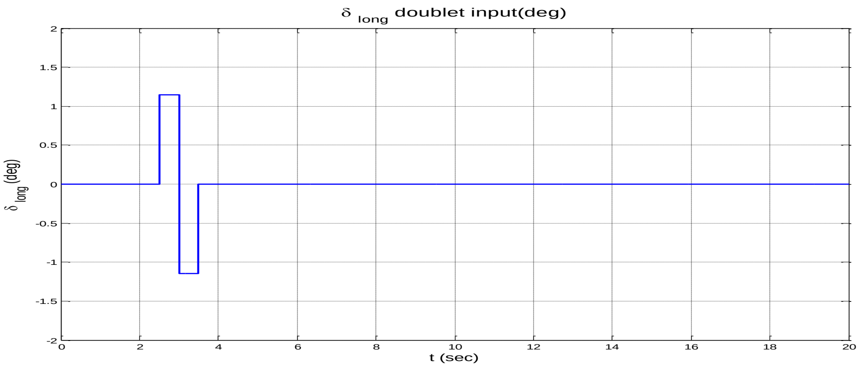

Figure 6: Longitudinal cyclic doublet input

To evaluate robustness of proposed LQR control, doublet input as shown in Figure 6 is used in simulation. The doublet input is actuated on the longitudinal deflection of the main rotor.

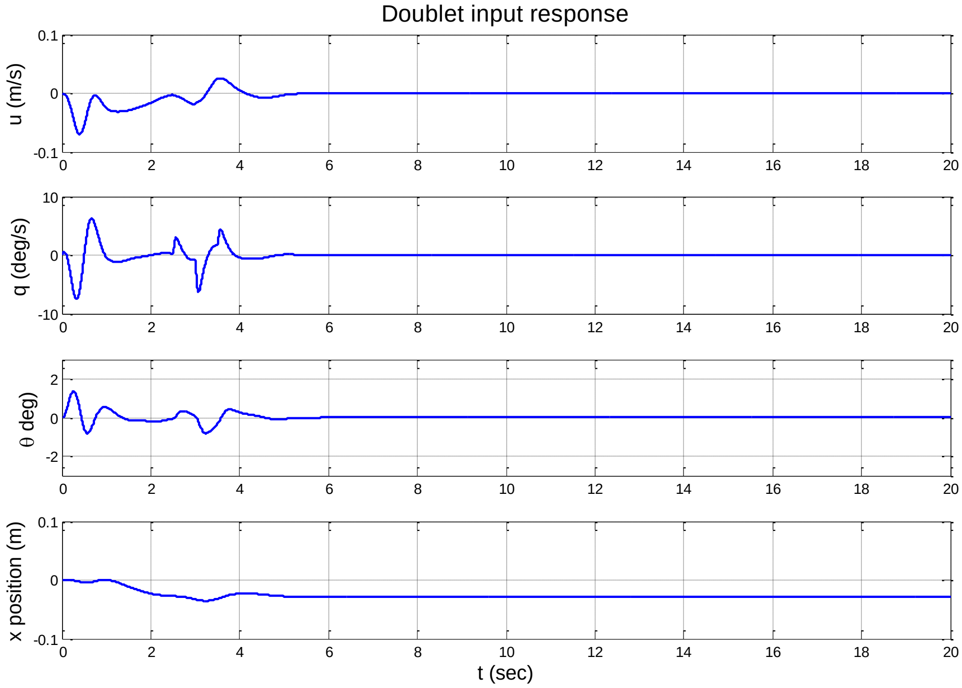

Figure 7: u, q, θ, x responses do to doublet input

Figure 7 show the response of the states: **u, q,** θ**, x** to the doublet input given. The simulation results reveal that the closed loop control system based on the LQR method designed at hover trim condition is quite robust to maintain at hover position.

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5. CONCLUSIONS

The feedback gain from design method has been simulated to be robust at hover flight condition. The minimum and maximum time constan are about 0.0065 second 0.377 second. Simulation results show the robustness of the closed-loop control system being disturbed by the doublet input on the longitudinal deflection of the main rotor, is stable although some drift occur on the translational X-Y-Z position. In the real implementation, limits in actuator dynamics should be carefully considered.

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