Prime Ideal and Weakly Prime Ideal The Endomorphism Ring of □□□Z(□□(R))

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Prime Ideal And Weakly Prime Ideal For The Endomorphism Ring of $\boldsymbol{End_{\mathbb{Z}}(M_n(\mathbb{R}))}$

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Abstract. The concepts of prime ideals and weakly prime ideals have been studied in the commutative ring. To extend the range of these two concepts, have introduced an extension of the prime ideal and weakly prime ideal concepts for rings that aren't necessarily commutative. Further, Nico [1] gives the concept of (a, b) *twin- zero* in weakly prime ideals in non-commutative ring, which is then used to show some properties in weakly prime ideals in non-commutative rings. A module endomorphism with additive and composition function is a non-commutative rings with identity. Therefore, in this study, research will be carried out on some properties of prime ideals and weakly prime ideals in ring endomorphism, especially at ring $End_{\mathbb{Z}}(M_n(\mathbb{R}))$.
Keywords: prime ideal, weakly prime ideal, endomorphism ring, non-commutative ring.

INTRODUCTION

The development of ring theory, especially ideals, has known the concept of prime ideals. A proper ideal I in ring R is called a prime ideal if every $a, b \in R$ with $ab \in I$ implies $a \in I$ or $b \in I$ [2, 3, 4, 5, 6]. The notion of the prime ideal has played an essential role in studying commutative rings. To extend the scope of the study of prime ideals, Neal[7] further explains his idea of prime ideals in the concept of a ring that is not necessarily commutative. The concept is that for $A, B \subseteq R$ are two right (left) ideals in R, with $AB \subseteq I$ implying $A \subseteq I$ or $B \subseteq I$. Anderson and Smith [8] further generalized the concept of the prime ideal in the commutative ring, that the proper ideal I in ring R is called a weakly prime ideal if every $a, b \in R$ with $0 \neq ab \in I$ imply $a \in I$ or $b \in I$. Yasuyuki et al. [9] further extend the concept into weakly prime in non-commutative ring. The concept is that for $A, B \subseteq R$ are two right (left) ideals in R, with $0 \neq AB \subseteq I$ implying $A \subseteq I$ or $B \subseteq I$. Furthermore, can be investigated the properties of prime ideals and weak prime ideals in a special non-commutative ring. So

Furthermore, with R a commutative ring with identity, M a commutative group, and a scalar multiplication operations, can be formed an R-module M. The relationship of the two modules is further introduced in the concept of module homomorphism. A mapping f that maps from the R-module M to the R-module M' is called a module homomorphism if f preserves the addition and multiplication of the scalar operations on R-module M to the addition and multiplication operations of the scalars on R-module M'. Then if $M = M'$, f is called a module endomorphism. The set of all R-module endomorphisms is denoted by $End_R(M)$. The set $End_R(M)$ with additive and composition function is a non-commutative ring with identity, that identity i.e., $f(x) = x$. Such this ring is called an endomorphism ring [4]. Given the ring of $\mathbb{Z}$ and the set of n square matrices over the real number denoted by $M_n(\mathbb{R})$. Since $\mathbb{Z}$ is a ring and $M_n(\mathbb{R})$ is a commutative group, we have that $M_n(\mathbb{R})$

as a $\mathbb{Z}$-module. From $\mathbb{Z}$-module $M_n(\mathbb{R})$, we can construct $End_{\mathbb{Z}}(M_n(\mathbb{R}))$. Have known that $(End_{\mathbb{Z}}(M_n(\mathbb{R})), +, \circ)$ is a non-commutative ring, such that we will study some properties of prime ideals and weakly prime ideals in the $(End_{\mathbb{Z}}(M_n(\mathbb{R})), +, \circ)$. Further $(End_{\mathbb{Z}}(M_n(\mathbb{R})), +, \circ)$ can simply be written with $End_{\mathbb{Z}}(M_n(\mathbb{R}))$.

PRIME IDEAL AND WEAKLY PRIME IDEAL OF RING $End_{\mathbb{Z}}(M_n(\mathbb{R}))$

In this subsection, the results of the research obtained will be discussed. To start the discussion, we will first give an example that the composition of the function on $End_{\mathbb{Z}}(M_n(\mathbb{R}))$ does not fulfill the commutative property, so ring $End_{\mathbb{Z}}(M_n(\mathbb{R}))$ is a non-commutative ring.

Example 1. Let $M_n(\mathbb{R})$ be a $\mathbb{Z}$-module. Given $k \in \mathbb{Z}$, $f\left(\begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix}\right) = \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix}$ and $g\left(\begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix}\right) = \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ for any $\begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix} \in M_n(\mathbb{R})$. Notice that:

1. $$\begin{aligned} f\left(\begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix} + \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}\right) &= \left(\begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix} + \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}\right) \cdot \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix} \\ &= \begin{bmatrix} x_{11} + y_{11} & x_{12} + y_{12} \\ x_{21} + y_{21} & x_{22} + y_{22} \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix} \\ &= \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix}, z_{ij} = x_{ij} + y_{ij}, i = 1,2, j = 1,2 \\ &= \begin{bmatrix} z_{11} + z_{12} & 3z_{11} + 2z_{12} \\ z_{21} + z_{22} & 3z_{21} + 2z_{22} \end{bmatrix} \\ f\left(\begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix}\right) + f\left(\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}\right) &= \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix} \\ &= \begin{bmatrix} x_{11} + x_{12} & 3x_{11} + 2x_{12} \\ x_{21} + x_{22} & 3x_{21} + 2x_{22} \end{bmatrix} + \begin{bmatrix} y_{11} + y_{12} & 3y_{11} + 2y_{12} \\ y_{21} + y_{22} & 3y_{21} + 2y_{22} \end{bmatrix} \\ &= \begin{bmatrix} x_{11} + y_{11} + x_{12} + y_{12} & 3x_{11} + 3y_{11} + 2x_{12} + 2y_{12} \\ x_{21} + y_{21} + x_{22} + y_{22} & 3x_{21} + 3y_{21} + 2x_{22} + 2y_{22} \end{bmatrix} \\ &= \begin{bmatrix} z_{11} + z_{12} & 3z_{11} + 2z_{12} \\ z_{21} + z_{22} & 3z_{21} + 2z_{22} \end{bmatrix}, z_{ij} = x_{ij} + y_{ij}, i = 1,2, j = 1,2 \end{aligned}$$

   So, $f\left(\begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix} + \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}\right) = f\left(\begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix}\right) + f\left(\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}\right)$

2. $$\begin{aligned} f\left(k \cdot \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix}\right) &= \left(k \cdot \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix}\right) \cdot \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix} \\ &= \left(\begin{bmatrix} kx_{11} & kx_{12} \\ kx_{21} & kx_{22} \end{bmatrix}\right) \cdot \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix} \\ &= \begin{bmatrix} kx_{11} + kx_{12} & 3kx_{11} + 2kx_{12} \\ kx_{21} + kx_{22} & 3kx_{21} + 2kx_{22} \end{bmatrix} \\ k \cdot f\left(\begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix}\right) &= k \cdot \left(\begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix}\right) \\ &= k \cdot \begin{bmatrix} x_{11} + x_{12} & 3x_{11} + 2x_{12} \\ x_{21} + x_{22} & 3x_{21} + 2x_{22} \end{bmatrix} \\ &= \begin{bmatrix} kx_{11} + kx_{12} & 3kx_{11} + 2kx_{12} \\ kx_{21} + kx_{22} & 3kx_{21} + 2kx_{22} \end{bmatrix} \end{aligned}$$

   So, $f\left(k \cdot \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix}\right) = k \cdot f\left(\begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix}\right)$

Based on properties 1 and 2, then f is a module homomorphism and therefore f maps $M_n(\mathbb{R})$ to itself, then $f \in End_{\mathbb{Z}}(M_n(\mathbb{R}))$. In the same way, it can be shown that $g \in End_{\mathbb{Z}}(M_n(\mathbb{R}))$. Notice that:

$$f \circ g\left(\begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix}\right) = f\left(g\left(\begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix}\right)\right)$$

$$
\begin{aligned}
&= f\left(\begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\right) \\
&= \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5x_{11} + 3x_{12} & 12x_{11} + 7x_{12} \\ 5x_{21} + 3x_{22} & 12x_{21} + 7x_{22} \end{bmatrix} \\
g \circ f\left(\begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix}\right) &= g\left(f\left(\begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix}\right)\right) \\
&= g\left(\begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix}\right) \\
&= \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5x_{11} + 4x_{12} & 9x_{11} + 7x_{12} \\ 5x_{21} + 4x_{22} & 9x_{21} + 7x_{22} \end{bmatrix}
\end{aligned}
$$

Since the multiplication of two matrices is not always commutative, and based the result of example, then $f \circ g\left(\begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix}\right) \neq g \circ f\left(\begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix}\right)$. Based this case, then we know that $End_{\mathbb{Z}}(M_n(\mathbb{R}))$ is a non-commutative ring.

Since the $End_{\mathbb{Z}}(M_n(\mathbb{R}))$ is a non-commutative ring, this article uses the concept of prime ideal and weakly prime ideal on non-commutative rings. Furthermore, the ideals used are right ideals. Examples of prime ideal on $End_{\mathbb{Z}}(M_n(\mathbb{R}))$ is given in the following example.

Example 2. Let $f(A) = A$ for any $A \in M_n(\mathbb{R})$, it can be checked that $f \in End_{\mathbb{Z}}(M_n(\mathbb{R}))$. By f, can be constructed a principal right ideal, i.e.,

$$
\begin{aligned}
F = \langle f \rangle &= \{f \circ g(A) | f(A) = A, g \in End_{\mathbb{Z}}(M_n(\mathbb{R})), A \in M_n(\mathbb{R})\} \\
&= \{f(g(A)) | f(A) = A, g \in End_{\mathbb{Z}}(M_n(\mathbb{R})), A \in M_n(\mathbb{R})\}, \text{ since } g \in End_{\mathbb{Z}}(M_n(\mathbb{R})), \text{ so} \\
&= \{f(B) | f(B) = B, B = g(A) \in M_n(\mathbb{R})\}
\end{aligned}
$$

Notice that for each G, H the right ideals on $End_{\mathbb{Z}}(M_n(\mathbb{R}))$. Since G and H are two right ideals on $End_{\mathbb{Z}}(M_n(\mathbb{R}))$, the product of the two ideals is explained as follows.

$$
\begin{aligned}
G \circ H &= \{g \circ h | g \in G, h \in H\} \\
&= \{g(h(A)) | g \in G, h \in H, A \in M_n(\mathbb{R})\}
\end{aligned}
$$

If $G \circ H \subseteq F$, then $g(h(A)) = g(B) \in F$ for any $B \in M_n(\mathbb{R})$. Since $g(B) \in F$ for any $g \in G$, then $G \subseteq F$. So, based on the definition of prime ideal on non-commutative ring, F is a prime ideal on $End_{\mathbb{Z}}(M_n(\mathbb{R}))$.

Example 3. For any $\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \in M_n(\mathbb{R})$, $g\left(\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}\right) = \begin{bmatrix} a_{11} & 0 & \cdots & 0 \\ 0 & a_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a_{nn} \end{bmatrix}$, it can be checked that $g \in End_{\mathbb{Z}}(M_n(\mathbb{R}))$. By g, can be constructed a principal right ideal, i.e.,

$$
\begin{aligned}
G = \langle g \rangle &= \left\{ g \circ h(A) | g\left(\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}\right) = \begin{bmatrix} a_{11} & 0 & \cdots & 0 \\ 0 & a_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a_{nn} \end{bmatrix}, h \in End_{\mathbb{Z}}(M_n(\mathbb{R})), A \in M_n(\mathbb{R}) \right\} \\
&= \left\{ g(h(A)) | g\left(\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}\right) = \begin{bmatrix} a_{11} & 0 & \cdots & 0 \\ 0 & a_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a_{nn} \end{bmatrix}, h \in End_{\mathbb{Z}}(M_n(\mathbb{R})), A \in M_n(\mathbb{R}) \right\},
\end{aligned}
$$

since $h \in End_{\mathbb{Z}}(M_n(\mathbb{R}))$, so

$$= \left\{ g\left(\begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nn} \end{bmatrix}\right) \middle| g\left(\begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nn} \end{bmatrix}\right) = \begin{bmatrix} b_{11} & 0 & \cdots & 0 \\ 0 & b_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & b_{nn} \end{bmatrix}, \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nn} \end{bmatrix} = h(A) \in M_n(\mathbb{R}) \right\}$$

In the same way with F, it can be shown that G is a prime ideal on $End_{\mathbb{Z}}(M_n(\mathbb{R}))$.

Based Example 2 and Example 3, we can generalize the concept in the following proposition.

Proposition 4. *Every principal right ideals in $End_{\mathbb{Z}}(M_n(\mathbb{R}))$ is a prime ideal in $End_{\mathbb{Z}}(M_n(\mathbb{R}))$.*

Proof. Let F is a principal right ideal in $End_{\mathbb{Z}}(M_n(\mathbb{R}))$. For any $G, H \in End_{\mathbb{Z}}(M_n(\mathbb{R}))$, with $G \circ H \subseteq F$. Therefore, for any $g \in G$ and $h \in H$, then $g \circ h \in F$. Furthermore, $g \circ h = g(h(A)) \in F$ for any $A \in M_n(\mathbb{R})$, imply that $g \in F$. So, F is a prime ideal in $End_{\mathbb{Z}}(M_n(\mathbb{R}))$.

Since the weakly prime ideal is a generalization of the prime ideal, then every prime ideal is a weakly prime ideal. This means that every principal right ideals in $End_{\mathbb{Z}}(M_n(\mathbb{R}))$ is a weakly prime ideal in $End_{\mathbb{Z}}(M_n(\mathbb{R}))$. Next, we will give some properties of prime ideal and weakly prime ideal in $End_{\mathbb{Z}}(M_n(\mathbb{R}))$. Neal [7] in Lemma 2 explain that if A is an ideal and P is a prime ideal in R, then $A \cap P$ is a prime ideal in the ring A. Based this concept, the following proposition is obtained.

Proposition 5. *If F and G are principal right ideals in $End_{\mathbb{Z}}(M_n(\mathbb{R}))$, then $F \cap G$ is a prime ideal in the F and prime ideal in G.*

Proof. Based on the concept of intersection between ideal and prime ideal by Neal [7], it is found that F can be seen as ideal, and G can be seen as prime ideal or vice versa. Therefore, $F \cap G$ is a prime ideal in F and a prime ideal in G.

Further, before the weakly prime's propertie is given, Nico [1] provides the following definition of *twin-zero*.

Definition 6. *Let I be a weakly prime ideal in R. We say (a, b) is a twin-zero of I if $\langle a\rangle\langle b\rangle = 0_R, a \notin I$ and $b \notin I$.*

By Example 2 and Example 3, we have prime ideals in $End_{\mathbb{Z}}(M_n(\mathbb{R}))$. Next, the following *twin-zero* example is given.

Example 7. Let F (i.e., F in Example 2) be weakly prime ideal in $End_{\mathbb{Z}}(M_n(\mathbb{R}))$. The $f(A) = 0_n$ with $A \in M_n(\mathbb{R})$, and any $g \in End_{\mathbb{Z}}(M_n(\mathbb{R}))\backslash F$. We have (f, g) is twin zero in F. This is because $f, g \notin F$, and

$$\begin{aligned} \langle f\rangle \circ \langle g\rangle &= \{f \circ g(A) | A \in M_n(\mathbb{R})\} \\ &= \{f(g(A)) \,|A \in M_n(\mathbb{R})\} \\ &= \{f(B)\,| f(B) = 0_n, B \in M_n(\mathbb{R})\} \end{aligned}$$

Furthermore, a propertie of weakly prime ideal by Nico [1] in Lemma 1.5 is for any I a weakly prime ideal in R, and suppose that (a, b) is a *twin-zero* of I for some $\mathrm{a}, \mathrm{b} \in \mathrm{R}$. Then $\langle \mathrm{a}\rangle\mathrm{I} = \mathrm{I}\langle \mathrm{b}\rangle = 0$. Based this concept, the following proposition is obtained.

Proposition 8. *Let F be a principal right ideal in* $\mathrm{End}_{\mathbb{Z}}(\mathrm{M_n}(\mathbb{R}))$, $\mathrm{f(A)} = 0_{\mathrm{n}}$ *with* $\mathrm{A} \in \mathrm{M_n}(\mathbb{R})$, *and any* $g \in End_{\mathbb{Z}}(M_n(\mathbb{R}))\backslash F$. *Then* $\langle \mathrm{f} \rangle \circ \mathrm{F} = \mathrm{F} \circ \langle \mathrm{g} \rangle = 0$.

Proof. We have (f, g) is a twin zero in F. Since F is prime ideal (Based on Proposition 4), then F is a weakly prime ideal. Based on the ideal characteristics of weakly prime described by Nico [1], then $\langle \mathrm{f} \rangle \circ \mathrm{F} = \mathrm{F} \circ \langle \mathrm{g} \rangle = 0$.

CONCLUSIONS

By the theory and the ring we have i.e., $End_{\mathbb{Z}}(M_n(\mathbb{R}))$, we get some conclusions. The first is, $End_{\mathbb{Z}}(M_n(\mathbb{R}))$ is non-commutative ring. The second one, every principal right ideal in $End_{\mathbb{Z}}(M_n(\mathbb{R}))$ is a prime ideal and weakly prime ideal in $End_{\mathbb{Z}}(M_n(\mathbb{R}))$. Then, we have propositions i.e., that every intersection of two principal right ideal in $End_{\mathbb{Z}}(M_n(\mathbb{R}))$ is a prime ideal in one of them. The last result is a proposition that the product of principal right ideal genetrated by $f(A) = 0_n$ (i.e., $\langle \mathrm{f} \rangle$) and any principal right ideal $\langle \mathrm{g} \rangle$ equal with $0_{End_{\mathbb{Z}}(M_n(\mathbb{R}))}$.

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GRADEMARK REPORT

FINAL GRADE

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GENERAL COMMENTS

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