

A comparison of Heuristic method and Llewellyn's rules for identification of redundant constraints

by R. Heru Tjahjana

Submission date: 19-Nov-2020 05:59PM (UTC+0700)

Submission ID: 1450998266

File name: lewellyn_s_rules_for_identification_of_redundant_constraints.pdf (879.96K)

Word count: 2182

Character count: 11769

PAPER • OPEN ACCESS

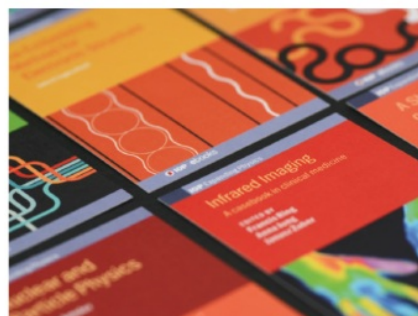
A comparison of Heuristic method and Llewellyn's rules for identification of redundant constraints

To cite this article: Y Estiningsih *et al* 2018 *J. Phys.: Conf. Ser.* **983** 012083

View the [article online](#) for updates and enhancements.

Related content

- [Mobility and Position Error Analysis of a Complex Planar Mechanism with Redundant Constraints](#)
- [An Improved Heuristic Method for Subgraph Isomorphism Problem](#)
- [Inventory control of raw material using silver meal heuristic method in PR. Trubus Alami Malang](#)



IOP ebooks™

Bringing together innovative digital publishing with leading authors from the global scientific community.

Start exploring the collection—download the first chapter of every title for free.

A comparison of Heuristic method and Llewellyn's rules for identification of redundant constraints

Y Estiningsih^{1,*}, Farikhin² and R H Tjahjana³

¹Magister Program of Mathematics, Departement of Mathematics, Faculty of Science and Mathematics, Diponegoro University, Jl. Prof Soedarto SH, Semarang 50275, Central Java, Indonesia.

^{2,3}Departement of Mathematics, Faculty of Science and Mathematics, Diponegoro University, Jl. Prof Soedarto SH, Semarang 50275, Central Java, Indonesia.

*Corresponding author: estiningsih10@gmail.com

Abstract. Important techniques in linear programming is modelling and solving practical optimization. Redundant constraints are consider for their effects on general linear programming problems. Identification and reduce redundant constraints are for avoidance of all the calculations associated when solving an associated linear programming problems. Many researchers have been proposed for identification redundant constraints. This paper a comparison of Heuristic method and Llewellyn's rules for identification of redundant constraints.

1. Introduction

Large-scale linear programming problems is extremely important for different applications in technology and economics [1]. Interior point and simplex are methods that are used for solving large – scale linear programming problems [2]. In fact, large-scale linear programming problems almost always contain a significant number of redundant constraints and variables [3]. One of the most effect on general linear programming problems is redundant constraints. A constraint is said to be redundant if constraint does not change the feasible region that is constituted by this set [4]. If feasible region is empty then the system is inconsistent, the linear programming problems have no solution [5]. Redundant constraints might seem that substantial redundancy could only arise from through poor formulation linear programming problems. Duplicate rows are multiple of row from other rows. Duplicate rows in the same problems are redundant constraints. Reduce redundant constraints in linear programming problems to avoid obviously infeasible models [2]. Detecting and removing redundancy also important in linear programming problems are the avoidance of all the calculations associated with those constraints when solving problems [6].

Many researchers have proposed different methods for identify redundant constraints and reduce linear programming problems. Paulraj et al. proposed Heuristic constraints by using the intercept matrix of constraints of a linear programming problem [7]. Brearly *et al.* proposed a method which requires the lower and upper bounds of the variable. If the lower and upper bounds have a far space, redundant constraints cannot be identified. Stojkovic and Staminirovic proposed a method to identify redundant constraints by applying the maximum and minimum principle. Stojkovic and Staminirovic used objective function and constraints in linear programming problems [8]. Llewellyn's rules we used



to identify redundant constraints using 4 rules. However Llewellyn's rules only compare two constraints in linear programming problems. If many constraints are employed will take place many steps and inconsisten conclusion redundant or non-redundant [9].

In this Heuristic method, the dominance property of the intercept matrix of constraints is exploited to reduce the search space to find the optimal or near optimal solution. Heuristic method is used to identify the redundant constraints of linear programming problem before solving it. [10]. Paulraj et al. proposed Heuristic method which use the intercept matrix of constraints in a linear programming problems [7]. The state space simulation Heuristic can be asily incorporated into the increment and iterative approaches, and used to generate new constraints. The intuition behind this Heuristic is that the regions that are visited more likely are also more important for the quality of the solution. Heuristic constraint generation may help us to identify redundant constraint [11].

Identification redundant constraints which use algebraic rules to simplifications and reductions of a linear programming are Llewellyn's rules. Llewellyn's rules are used to identify some spesial redundant constraints. Llewellyn's rules are intended to identify redundant constraints by the presence of one other constraint and all non-negative variables of constraints [9].

In the paper, we discuss comparison conducted for between Heuristic method and Llewellyn's rules. There are, we in this study discussed comparison Heuristic method and Llewellyn's rules to identify redundant constraints in linear programming.

2. Methods

Consider the following system of m nonnegative linear inequality in n variable with $\leq m$:

$$\begin{aligned} \mathbf{Ax} &\leq \mathbf{b} \\ \mathbf{x} &\geq \mathbf{0}. \end{aligned} \quad (2.1)$$

where matrix $\mathbf{A} = [a_{ij}]_{\substack{1 \leq i \leq m \\ 1 \leq j \leq n}} \in R^{m \times n}$, vector $\mathbf{b} = [b_i]_{1 \leq i \leq m} \in R^m$, vector $\mathbf{x} = [x_j]_{1 \leq j \leq n} \in R^n$, null vector $\mathbf{0} \in R^n$.

Let $\mathbf{Ax} \leq \mathbf{b}$ be constraints of the system (2.1) and let $S = \{\mathbf{x} \in R^n \mid \mathbf{Ax} \leq \mathbf{b}, \mathbf{x} \geq \mathbf{0}\}$ be the feasible region associated with system (2.1). Let S_k be feasible region associated with the system of inequality $\mathbf{A}_i \mathbf{x} \leq b_i, i = 1, \dots, m, i \neq k$ where $\mathbf{A}_i = [a_{ij}]_{\substack{1 \leq i \leq m \\ 1 \leq j \leq n}} \in R^{m \times n}$ is as follow,

$$S_k = \left\{ \mathbf{x} \in R^n \mid \begin{cases} \sum_{j=1}^n a_{ij} x_j \leq b_i, i = 1, \dots, m, i \neq k \\ x_j \geq 0, j = 1, \dots, n \end{cases} \right\} \quad (2.2)$$

Two important problems in linear programming problems are, the first is construction modelling linear program and, the second is eliminate redundant constraints [12]. Redundant constraints in the system (2.1) do not change feasible region S . Redundant constraints can be classified, which are weakly redundant and strongly redundant constraints . There kinds of redundants describe as follows,

Definition 2.1. [8]

Given constraint $\sum_{j=1}^n a_{kj} x_j \leq b_k$ in the system inequality (2.1),

1. Redundant if $S = S_k$.
2. Weakly redundant if $S_k = \emptyset$.
3. Strongly redundant if $S_k > \emptyset$.

The form of linear programming problem is as follows,

$$\begin{aligned} \text{Linear Programming} \quad & \max z = \mathbf{cx} \\ & \mathbf{Ax} \leq \mathbf{b} \\ & \mathbf{x} \geq \mathbf{0}. \end{aligned} \quad (2.3)$$

Where matrix $\mathbf{A} = [a_{ij}]_{\substack{1 \leq i \leq m \\ 1 \leq j \leq n}} \in R^{m \times n}$, vector $\mathbf{b} = [b_i]_{1 \leq i \leq m} \in R^m$, vector $\mathbf{x} = [x_j]_{1 \leq j \leq n} \in R^n$, null vector $\mathbf{0} \in R^n$, dan vector $\mathbf{c} = [c_j]_{1 \leq j \leq n} \in R^n$.

Now we write the established algorithm detect redundant constraints. The methods are Heuristic method and Llewellyn's rules,

Algorithm 1 [7,8] (Heuristic method)

Input :
 Linear Programming $\max z = \mathbf{c}\mathbf{x}$
 $\mathbf{A}\mathbf{x} \leq \mathbf{b}$
 $\mathbf{x} \geq \mathbf{0}$.

Output :
 Linear programming without redundant constraints.

1. Let I is the index related to basic variables (slack variables), with $I = [1, 2, \dots, m]$, while J is an index related to the decision variables, with $J = [1, 2, \dots, n]$.
2. Intercept Matrix $\theta = [\theta_{ji}]$ with each element of the matrix as follows,

$$\theta_{ji} = \frac{b_i}{a_{ij}}: a_{ij} > 0 \text{ for } j \in J, i \in I.$$

3. Calculate $z_j - c_j$ for all non-basic variables.
4. Calculated $\beta_j = \min\{\theta_{ji}\}$ for $j \in J, i \in I$.
5. Calculated $z'_j - c'_j = \beta_j(z_j - c_j)$ for $j \in J$.
6. Select $z'_k - c'_k = \min\{z'_j - c'_j\}$.
7. If $z'_k - c'_k \geq 0$, it has no redundant constraints and stopped. Otherwise, subtract index k element of J ($J = J - \{k\}$).
8. Select $\theta_{kl} = \min\{\theta_{kl}\} = \beta_k$, subtract index l element of I ($I = I - \{l\}$) and find p such that $\min\{\theta_{pl}\} = \beta_p$ for $p \in J$, so $J = J - \{p\}$.
9. If $J = \{\emptyset\}$, then go to step 5. Otherwise, go to step 3.
10. If $I = \{\emptyset\}$, then it does not have redundant constraints and stopped. Otherwise, $i \in I$ as redundant constraints and stopped.

Algorithm 2 [9] (Llewellyn's Rules)

Input :
 Linear Programming $\max z = \mathbf{c}\mathbf{x}$
 $\mathbf{A}\mathbf{x} \leq \mathbf{b}$
 $\mathbf{x} \geq \mathbf{0}$.

Output :
 Linear programming without redundant constraints.

Let I is the index related to basic variables (slack variables), with $I = [1, 2, \dots, m]$, while J is an index related to the decision variables, with $J = [1, 2, \dots, n]$.

Rule 1: Given two inequalities with b_k and $b_s \geq 0, \forall k, s \in I$, and $k \neq s$, if

$$\frac{b_k}{a_{kj}} \geq \frac{b_s}{a_{sj}}, \forall j \in J \text{ then the } k\text{th constraints is redundant.}$$

Rule 2: Given two inequalities with b_k and $b_s < 0, \forall k, s \in I$, and $k \neq s$, if

$$\frac{b_k}{a_{kj}} \leq \frac{b_s}{a_{sj}}, \forall j \in J \text{ then the } k\text{th constraints is redundant.}$$

3. Result and Discussion

In the section we give several examples linear programming which there established method have not overcome for detecting redundant constraints,

Example 1

$$\max 3x_1 + 5x_2$$

$$x_1 + x_2 \leq 5$$

$$\begin{aligned}
 6x_1 + 4x_2 &\leq 24 \\
 6x_1 + 3x_2 &\leq 18 \\
 4x_1 + 6x_2 &\leq 24 \\
 x_1 + x_2 &\leq 4 \\
 x_1, x_2 &\geq 0.
 \end{aligned}$$

Example 2

$$\begin{aligned}
 \max 7x_1 + 5x_2 \\
 3x_1 + 2x_2 &\leq 6 \\
 2x_1 + 3x_2 &\leq 6 \\
 4x_1 + 2x_2 &\leq 8 \\
 3x_1 + 5x_2 &\leq 15 \\
 4x_1 + 6x_2 &\leq 24
 \end{aligned}$$

Example 3

$$\begin{aligned}
 \max 7x_1 + 5x_2 \\
 x_1 + x_2 &\leq 5 \\
 2x_1 + 3x_2 &\leq 6 \\
 3x_1 + 2x_2 &\leq 6 \\
 4x_1 + 2x_2 &\leq 8 \\
 2x_1 + 4x_2 &\leq 8 \\
 x_1, x_2 &\geq 0.
 \end{aligned}$$

In the Example 1, and Example 2 with 1 weakly redundant constraint, and 2 strongly redundant constraints. Meanwhile Example 3 with 2 weakly redundant constraint, and 1 strongly redundant constraints. The purpose of this examples are to show that Heuristic method can not detect weakly redundant constraints. Examples detecting redundant constraints of linear programming problems above can be served in Table 1.

Table 1. Examples of redundant constraints of linear programming problems.

No	Example	F	W	R	H	L
1	Example 1	2 {3,5}	1 {4}	3 {1,2,4}	2 {1,2}	3 {1,2,3}
2	Example 2	2 {1,2}	1 {3}	3 {3,4,5}	2 {4,5}	3 {3,4,5}
3	Example 3	2 {2,3}	2 {4,5}	3 {1,4,5}	1 {1}	3 {1,4,5}

Nomenclature:

- F : Number of constraints feasible region.
- W : Number of constraints on feasible region (weakly redundant constraint).
- R : Number of constraints redundant constraints.
- H : Number of constraints are detected by Heuristic method.
- L : Number of constraints are detected by Llewellyn's rules.

Consider Table 1. In the example 1 linear programming problem with 2 constraints forming the feasible region, 1 weakly redundant constraint, and 2 strongly redundant constraints, solved using Heuristic method only 2 strongly redundant constraints that can be identified and 1 weakly redundant constraints cannot be identified meanwhile using Llewellyn's rules can identify 3 redundant constraints are 2 strongly redundant constraints and 1 weakly redundant constraint. Example 2 and Example 3 are analogous to Example 1.

4. Conclusion

Heuristic method for identifying redundant constraints has been compared with Llewellyn's rules. Heuristic method can identify redundant constraints in linear programming problems without weakly redundant constraints. Llewellyn's rules may only be applied if all coefficients have the same signs.

Llewellyn's rules can identify all redundant constraints in linear programming problems based on term Llewellyn's rules. Using Llewellyn's rules on linear programming problems with many constraints will require more time and more work. This is because sometimes comparing with different constraints will lead to different conclusions. While Heuristic method identifies redundant constraints by rapidly with an iteration system. In the other hand, Llewellyn's rules are a better method than Heuristic method for identifying redundant constraints.

Further research comparing the Heuristic methods, Llewellyn's rule, and Stojkovic and Stanimirovic method. The three interesting methods for analyzing the characteristics of each method. All three methods use objective functional and constraints to identify redundant constraints.

References

- [1] Gutman P O and Ioslovich I 2007 *Automation and Remote Control* **68** 116
- [2] Meszaeros C and Suhl U H 2003 *OR Spectrum* **25** 575
- [3] Ioslovich I 2001 *Siam J Optim* **12** 262
- [4] Sumanthi P, Balachander R, Karpagam A, and Pugazhenth P 2015 *International Journal of Scientific & Engineering Research* **6** 66
- [5] Yegnanarayanan V and Sreekumar S 2013 *J Math Comput Sci* **3** 1076
- [6] Sumanthi P, P Balachander P, Karpagam A and Pugazhenth P 2015 *International Journal of Scientific & Engineering Research* **6** 66
- [7] Paulraj S Chellappan C and Natesan T R 2006 *International Journal of Computer Mathematics* **83** 675
- [8] Paulraj S and Sumathi P 2010 *Journal Mathematical Problems in Engineering* **1** 1
- [9] Llewellyn R W 1979 *Zeitschrift fur Operations Research* **23** 197
- [10] Krishna V and Balachandar K S 2010 *International Journal of Computational and Mathematical Sciences* **4** 259
- [11] Kveton, Branislav and Hauskrecht M 2004 *American Association for Artificial Intelligence* **306** 1
- [12] Stojkovic, Nebojsa V and Stanimirovic P S 2001 *European Journal of Operational Research* **131** 417

A comparison of Heuristic method and Llewellyn's rules for identification of redundant constraints

ORIGINALITY REPORT

9%

SIMILARITY INDEX

6%

INTERNET SOURCES

3%

PUBLICATIONS

0%

STUDENT PAPERS

PRIMARY SOURCES

1

repository.lppm.unila.ac.id

Internet Source

5%

2

Current Developments in the Interface
Economics Econometrics Mathematics, 1982.

Publication

1%

3

Aydilek, Asiye, Harun Aydilek, and Ali
Allahverdi. "Production in a two-machine
flowshop scheduling environment with uncertain
processing and setup times to minimize
makespan", International Journal of Production
Research, 2015.

Publication

1%

4

C.-N. Lu, M.R. Unum. "Network constrained
security control using an interior point
algorithm", IEEE Transactions on Power
Systems, 1993

Publication

1%

5

Matthias Aschenbrenner, Lou van den Dries,
Joris van der Hoeven. "Asymptotic Differential

1%

Algebra and Model Theory of Transseries", Walter de Gruyter GmbH, 2017

Publication

6	www.professormartin.net Internet Source	<1%
7	arxiv.org Internet Source	<1%
8	info.cba.ksu.edu Internet Source	<1%

Exclude quotes Off

Exclude matches Off

Exclude bibliography Off