An analytic solution of uncoupled multi-agent model and its benefit through optimal control system with attractor and repellant

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Abstract. This paper gives the analytic solution of uncoupled multi-agent model. This solution describes the optimal path of each agent. In this exposition, optimal control approach is used to model uncoupled multi-agent swarm with attractor and repellant. The special functional cost contain repellent cost functional is used to guarantee each agent never collides one to the others. The attractor term in the special functional cost make each agent never mover far away to the others. This paper also gives the benefit if the agents move in the multi-agent system.

1. Introduction

In the recent years, also as motivation in paper writing, multi-agent appear in many real case. Multi-agent in natural phenomenon is recognized as swarm. Swarm is a natural phenomenon happening on groups of animals([1],[2],[3]). Swarm phenomenon is also happening on a group out of animals. For example, phenomenon happens on unmanned aerial vehicle, robots, and airplanes.

Peoples modeled this phenomenon for example, Breder [4] wrote model of animal aggregation with repulsive force and attractive force. Warburton and Lazarus [5] studied Tendency-distances model of social cohesion in animal group. Mogilner and Keshet [6] described continuum models for swarming behavior based on non-local interactions. The interactions are assumed to influence the velocity of the organism. The model consists of integral-differential advection-diffusion equations. Keshet [7] proposed mathematical models of swarming and social aggregation. He surveyed some of the problems connected with aggregation of social organism and indicate some mathematical and model challenges. In her paper, she modeled swarm phenomenon, but she did not use control approach. Gazi and Passino [8] provided a model of an aggregation swarm. They specify an "individual based" continuous time model for swarm in n-dimensional space and studied its stability properties. Topaz and Bertozzi [9] constructed a continuum for the motion of biological organisms experiencing social interactions and studied its pattern-forming behavior. The model toke form of a conservation law in two spatial dimensions. The social interactions are modeled in the velocity term, which is nonlocal in the population density and includes a parameter that controls the interaction length scale. Shi, Wang, and Chu [10] considered an anisotropic swarm model with an attraction/repulsion function and studied its aggregation properties. Pranoto,Tjahjana, and Muhammad [11],[12] wrote simulation of swarm modeling through optimal control and simulation of swarm modeling trough bilinear optimal control.

The examples of multi-agent utilization, the reader can consul in [13] and [14] for swarm of flying robot. The other paper which cover multi-agent in cooperation among flying robot the reader who interesting can read [15] and [16]. The other researches in flying robot as multi-agent control can be viewed in [17], [18] and [19]. If the reader want to know multi-agent system that the agents are quadrotor can reference [20] and [21]. The utilization of multi-agent in surveillance can be referenced in [22] and [23]. The research paper about the utilization of multi-agent in fuel saving can consult in [24] and [25]. Research papers ([26]-[33]) are the newest papers which exposed multi agent, but these papers did not consider analytic solution of multi agent.
Furthermore, since in some real cases the swarm leader does not exist, in this paper, we expose the model without swarm leader. The absence of the swarm leader is shown by the equity of the agents. Each agent cannot control the others. Since among the swarm agents, one and the others never collide, we put it in the cost functional. It penalizes all members if two or more members move too close to each other. We also put the attractor term to guarantee all members move away far to the others.
The main contribution of this paper, we propose analytic solution *a model of uncoupled multi-agent swarm system with attractor and repellant* using optimal control approach with special cost functional. The special cost functional which used in this paper is not standard cost functional as usually in optimal control course, but the cost functional contains repellant and attractor term. The analytic solution, physically can be viewed as an optimal path for each agent, so the next section will explore the optimal path. The other main contribution of this paper is a theorem for show that the control used in multi-agent is lower than agents move in solo.
Since optimal solution can be viewed as Optimal path then in the next section the optimal path will be discussed.

2. Optimal Path

In general, the path of the i-th agent is governed by the vector function $x_i(t)$ in $\mathbf{R}^n$. Each agent controls itself through piecewise-continuous control function $u_i(t)$. The initial formation of the swarm is described by the initial condition $x_i(0) = x^0$. The final formation of the swarm is described by the final condition $x_i(T) = x^1$. The cost is defined by $J = \int_0^T g(x_1, x_2, \dots, x_m) + h(u_1, u_2, \dots, u_m)\, dt$. We seek an optimal control $u_i \in \mathcal{U}$ such that the state of the system can be steered from x^o to x^1 in time T with the minimum value of J. The trajectory $x(t)$ related to the optimal control is the **optimal path**, and the corresponding value of J is the **optimal cost**. We first define an extra state variable x_0 by state equation $\dot{x}_0 = f(x,u)$,$x_0(0) = 0$. Thus, the cost is given by $J = x_0(T)$. We next introduce the extended state vector $\hat{x}$ of dimension $nm + 1$, whose components are $x_0, x_1, x_2, \dots, x_m$. If we define the extended vector $\hat{f}$ similarly, the state equations can be written in the form $\dot{\hat{x}} = \hat{f}(x,u)$. The **Hamiltonian** $H(x,p,u)$ is defined by $H = \hat{p}^t \dot{\hat{x}} = \sum_{i=1}^{n} p_i f_i$, where $\hat{p}$ is the extended co-state vector of dimension $m + 1$. Hamilton's equation $\dot{\hat{p}} = -\frac{\partial H}{\partial \hat{x}}$ and $\dot{x} = \frac{\partial H}{\partial \hat{p}}$. Since H does not depend on x_0, these equations can be written in form $\dot{p}_0 = 0$, $\dot{p}_i = -\frac{\partial H}{\partial \hat{x}_i}$, $i = 1,2,\dots,m$. The Pontryagin Maximum Principle can now be stated as follows. Suppose that the problem has an optimal solution with an optimal control u^*. Then the following conditions must hold

- $p_0 = 1$
- u^* is the control function for which $H(x,p,u)$ reaches its infimum for all u
- The co-state equations have a solution $\hat{p}*$ and the state equation a solution x^* which takes the value x^0 at $t = 0$ and x^1 at $t = T$
- The Hamiltonian function is constant along optimal trajectory and this constant is zero if the terminal is free, that is $H(x^*, \hat{p}*, u^*) =$ constant if T is fixed and $H(x^*, \hat{p}^*, u^*) = 0$ if T is free and positive.

After the optimal path explored in this paper, the multi-agent model and its analytic solution will be considered in the next section.

3. Uncoupled Multi-agent Swarm with Attractor and Repellant

The controlling problem of multi-agent is brought to optimal control problem. The group of agent's dynamic are modeled by many decoupled control systems as follows

$$\begin{aligned} \dot{x}_1(t) &= f_1(x_1(t), u_1(t)) \\ \dot{x}_2(t) &= f_2(x_2(t), u_2(t)) \\ &\vdots \\ \dot{x}_k(t) &= f_k(x_k(t), u_k(t)). \end{aligned} \tag{1}$$

Thus, the i-th agent is governed by $\dot{x}_i(t) = f(x_i(t), u_i(t))$. The initial and boundary conditions of the system are given by

$$x_i(0) = s_i, \qquad x_i(T) = q_i. \tag{2}$$

The common objective of the agents transformed in a cost functional defined as follows

$$J = \int_0^T h(u_1, u_2, \cdots, u_n) + g_1(x_1, x_2, \cdots, x_n) + g_2(x_1, x_2, \cdots, x_n)dt. \tag{3}$$

The cost functional J consists of three terms, namely g_1, g_2 and h. The term g_1 represents the fact that two agents or more cannot collide. In other words, the term g_1 contributes to increase the cost if two agents or more move close to each other. The term g_1 is called repellant term. The term g_2 represents the fact that two agents cannot move far to the other. The term g_2 is called attractor term. The term h represents the total cost of the controls or energy used by the group of agents.

In this paper, we consider the model of uncoupled multi-agent swarm with k members or agents as follows

$$\begin{aligned} \dot{x}_1 &= A_1 x_1 + B_1 u_1 \\ \dot{x}_2 &= A_2 x_2 + B_2 u_2 \\ &\vdots \\ \dot{x}_k &= A_k x_k + B_k u_k, \end{aligned} \tag{4}$$

for $i = 1, 2, \ldots, k$. Model (4) is special form from model (1). In the model (4), position of i-th agent are symbolized by x_i, the symbol $\dot{x}_i$ denote for differentiation x_i with respect to t, A_i and B_i are contants. Control of the i-th agent are symbolized by u_i. Since an agent just control it self and cannot control the other agent, so model (4) is uncoupled multi-agent model. The initial and boundary conditions are given as

$$x_i(0) = s_i, x_i(T) = q_i, i = 1, \ldots, k. \tag{5}$$

In (5), 0 is initial time, so $x_i(0)$ can be viewed as initial position the i-th agent, and T is fixed final time then $x_i(T)$ can be viewed as final position the i-th agent. Consider the model (4) and conditions (5), x_i is the position of the i-th swarm agent, s_i is the initial position of the i-th swarm agent, and q_i is the final position of the i-th swarm agent. Since in this paper we will use optimal control approach and the most important things in optimal control is cost functional then the special cost functional which used in this paper is defined by

$$J = -\frac{1}{2}\int_0^T \sum_{i=1}^{k} \delta u_i^2 + \sum_{i=1}^{k-1} \sum_{\substack{j=2 \\ j>i}}^{k} \frac{\gamma}{||x_i - x_j||^2} + \sum_{i=1}^{k-1} \sum_{\substack{j=2 \\ j>i}}^{k} \mu ||x_i - x_j||^2 dt. \tag{6}$$

The cost functional equation (6) is special form from (3). In cost functional equation (6), the first summand represents the cost of the controls used and δ is constant. The second summand represents the penalty if the members move too close to each other and γ is repellant constant. The third summand represents attractor function between different two members and μ is attractor constant. The solutions of the system of differential equations are $x_i(t)$, $i = 1, 2, \cdots, k$ and $x_i(t)$ describes an equation of optimum trajectory or optimum path of the i-th agent. The main problem in this paper is with initial and boundary conditions (5) we want to minimize cost functional which satisfies system (4). Since we are going to use Pontryagin Maximum Principle, so the minimization of cost functional is equivalent with maximize the negative of cost functional. It is the reason the cost functional (6) use negative term. By the Pontryagin Maximum Principle and the repellent cost, the path of the agents never collide. It is similar to the case of natural swarm phenomena where the agents never collide. Also the fact that an agent cannot control the others is described by the independent equation of $x_i(t)$. The agent depends on the other agents in collective duty. The independent equation of $x_i(t)$ means $x_i(t)$ is not influenced by $x_j(t)$. In most natural swarm phenomena, there is no leader. We translate the absence of the leader by the similarity of k state equations.

Theorem 3.1

Consider the multi-agent model in (3.4), with initial and boundary values which given in (3.5) and also maximize the cost functional which given in (3.6), then the control for the i-th agent is

$$u_i(t) = \frac{K_i exp(-A_i t) B_i}{\delta p_0}; i = 1, \ldots, k,$$

and the analytic solution of the muli-agent system is

$$x_i(t) = -\frac{(-s_i exp(-\sqrt{A_i}T) + q_i) exp(\sqrt{A_i}t)}{(-exp(\sqrt{A_i}T) + exp(-\sqrt{A_i}T)} + \frac{(q_i - exp(\sqrt{A_i}T)s_i)(-exp(\sqrt{A_i}t)}{-exp(-\sqrt{A_i}T) + exp(-\sqrt{A_i}T)}.$$

Proof: The Hamiltonian function of the system is

$$H = \sum_{i=1}^{k} p_i(A_i x_i + B_i ui) - \sum_{i=1}^{k} \frac{1}{2} \delta p_0 u_i^2 - \frac{1}{2} \gamma p_0 \sum_{i=1}^{k-1} \sum_{\substack{j=2 \\ j>i}}^{k} \frac{1}{||x_i - x_j||^2}$$
$$-\frac{1}{2} \mu p_0 \sum_{i=1}^{k-1} \sum_{\substack{j=2 \\ j>i}}^{k} ||x_i - x_j||^2$$

The Hamiltonian system is

$$\frac{\partial H}{\partial p_i} = \dot{x}_i = A_i x_i + B_i u_i; i = 1, \ldots, k. \tag{7}$$

$$\begin{aligned} \frac{\partial H}{\partial x_i} = \dot{-p_i} &= p_i A_i - \frac{1}{2} \gamma p_0 \frac{\partial}{\partial x_i} \left(\sum_{i=1}^{k-1} \sum_{j=2 j>i}^{k} \frac{1}{||x_i - x_j||^2}\right) \\ &- \frac{1}{2} \mu p_0 \frac{\partial}{\partial x_i} \left(\sum_{i=1}^{k-1} \sum_{j=2 j>i}^{k} ||x_i - x_j||^2\right). \end{aligned} \tag{8}$$

By Pontryagin Maximum Principle[34], the necessary condition such that the system reaches its extremum is

$$\frac{\partial H}{\partial u_i} = 0 = p_i B_i - \delta p_0 u_i; i = 1, \ldots, k \tag{9}$$

Consider (8) we have

$$-\dot{p}_1 - p_1 A_1 - \dot{p}_2 - p_2 A_2 - \ldots - \dot{p}_{k-1} - p_{k-1} A_{k-1} = \dot{p}_k + p_k A_k. \tag{10}$$

From (10) we get

$$-\dot{p}_1 - \dot{p}_2 - \ldots - \dot{p}_k = p_1 A_1 + p_2 A_2 + \ldots + p_k A_k. \tag{11}$$

Next, from equation (11), and we are going to find the analytic solution for $p_i(t)$ we have equations as follows

$$\dot{p}_i = -p_i A_i, \tag{12}$$

for $i = 1, 2, \ldots, k$. Then we conclude that

$$p_i(t) = K_i exp(-A_i t), \tag{13}$$

with $K_i; i = 1, 2, \ldots, k$ are constants. From (9) we obtain

$$u_i(t) = \frac{p_i(t) B_i}{\delta p_0}; i = 1, \ldots, k \tag{14}$$

consider (13) then (14) can be written as

$$u_i(t) = \frac{K_i exp(-A_i t) B_i}{\delta p_0}; i = 1, \ldots, k. \tag{15}$$

The equation (15) can be viewed as control which used by the i-th agent. Consider the equation (7) and (15),

$$\dot{x}_i = A_i x_i + B_i \frac{K_i exp(-A_i t) B_i}{\delta p_0}; i = 1, \ldots, k. \tag{16}$$

The solution of differential equation (16) with consider the initial and boundary conditions (5) is

$$\begin{aligned} x_i(t) = &-\frac{(-s_i exp(-\sqrt{A_i}T) + q_i) exp(\sqrt{A_i}t)}{(-exp(\sqrt{A_i}T) + exp(-\sqrt{A_i}T)} \\ &+ \frac{(q_i - exp(\sqrt{A_i}T)s_i)(-exp(\sqrt{A_i}t)}{-exp(-\sqrt{A_i}T) + exp(-\sqrt{A_i}T)} \end{aligned} \tag{17}$$

■

Theorem 3.1 above can be interpreted as optimal trajectory determination for each agent. The result in (17) can be viewed as optimal trajectory equation of the i-th agent.

Benefit of Move as Multi-agent System

Consider the special cost functional in equation (6), especially in the first term which represents the cost of the controls used. One of the benefit of move as multi-agent is reduce of control cost. The control cost if the agents move in multi-agent, follows (6) is

$$\frac{1}{2}\delta \sum_{i=1}^{k} (u_i)^2. \tag{18}$$

Follow (6), if the agents move in solo then the control cost is

$$\frac{1}{2}\delta \left(\sum_{i=1}^{k} u_i\right)^2. \tag{19}$$

Theorem 4.1

The control cost in move together is lower than move in solo

Proof: Consider (18) and (19) above, through Cauchy-Schwarz inequality we get (18) less than or equal with (19). So in notation can be stated that $\sum_{i=1}^{k} (u_i)^2 \leqslant (\sum_{i=1}^{k} u_i)^2$, in other word is concluded that The control cost in move together is lower than move in solo. ■

5. Conclusion and Future Work

Multi-agent swarm modeling is interesting, because through this process, multi-agent swarm system can be controlled more effective. Swarm multi-agent is a behavioral metaphor for solving distributed problems it is based on the principles underlying the behavior of natural systems of many agents. The abilities of such systems appear to transcend the abilities of the constituent individual agents; in all the biological cases studied so far, the emergence of high-level control has been found to be mediated by nothing more than a small set of swarm multi-agent low-level interactions among individuals, and between individuals and the environment. One way to analyze and understand underlying common principles of swarm systems is to capture their dynamics at more abstract levels. Modeling is a means for saving time, enabling generalization to different platforms, and estimating optimal system parameters.

After modeling process, the important step is to find the solution of the model. Mathematically, if the analytic solution from the model is founded, we can make many simulation for the model. Analytic solution is general form equation which satisfy the given requirements. In this paper, the analytic solution of the model can be found succesfully. The analytic solution of uncoupled multi-agent swarm model with k members or agents which described in (4) can be shown in (17). In the future work, if possible, we will try to find analytic solution model multi-agent swarm phenomenon through non-linear optimal control with attractor and repellant. If the analytic solution for non linear case is impossible to find, the solution with approximation will be used to get optimal solution.

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