Global stability for linear system and controllability for nonlinear system in the dynamics model of diabetics population

by R. Heru Tjahjana

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A H Permatasari[1,*], R H Tjahjana[2] and T Udjiani[3]

[1,2,3]*Department of Mathematics, Faculty of Science and Mathematics Diponegoro University*
Jl. Prof. H. Soedarto, S.H. Tembalang, Semarang, Indonesia

E-mail: henindya23@gmail.com

Abstract. This paper presents global stability for linear system and controllability for nonlinear system in dynamic model. We discuss dynamic model of diabetics population which is autonomous linear system. The model considers the development of individual from a healthy stage to a pre-diabetic stage, then stage of diabetes without complication, to the stage of diabetes with complication and diabetics become disabled. In the linear model is investigated global stability using quadratic forms of Lyapunov function. After investigating the global stability of the linear system, a nonlinear optimal control model is established. Control variable in the model is the prevention effort to reduce the number of pre-diabetic into diabetic without and with complication. Controllability of nonlinear control system is investigated using Lie Bracket method. The results show that the linear system is globally asymptotically stable and the nonlinear system is locally accessible.

Keywords: Lyapunov, global stability, controllability, Lie Bracket, diabetics populations

1. Introduction

Mathematical models are widely used in solving the daily life problems. For example, problems involving many complex parameters and arranged in one system known as dynamic system. A dynamic system is a system of equations that is affected by changes in motion and time. Furthermore, this dynamic system equation is often transformed into a state-space equation. One important study in the dynamic system problem is investigate the condition of the system, whether the system is a stable or unstable and controllable or uncontrollable. A good dynamic system must be stable and controllable. These conditions are needed to reduce errors in the system due to disturbance so that the system can represent the real problem.

In this paper we present a dynamic model of diabetic population following previous mathematical models on diabetes [1-3]. The model illustrates the development of the diabetic population from healthy stage until disabled stage due to complications We establish model with and without control. Model without control is called linear system and model with control is called nonlinear system. The main purposes are to investigate the global stability of linear system and controllability of the nonlinear system. Stability problem can be solved by Lyapunov method and controllability is investigated using Lie brackets.

2. Research Method

The method used in this study is literature review and reference collection of theories that support the completion of this research. We collect references about the characteristics of diabetes, the causes of diabetes, diabetes complications, the treatments such as prevention, dynamical model of diabetes, and optimal control problem. From the diabetics population, we establish a dynamic model with and without control. We can prove the stability of linear system (model without control) and controllability of nonlinear control model. Therefore, the literature review aims to determine the use of the Lie Bracket to investigate the controllability of the nonlinear control model. Stability of linear system is investigated using Lyapunov function. In addition, we give numerical example so that we can understand the theorems used to investigate the stability and controllability of dynamic model.

3. Results and Discussion

3.1. Formulation of the Model

In this paper, the mathematical model constructed on the population of diabetics. Let $H = H(t), E = E(t), D = D(t), C = C(t)$ and $B = B(t)$ be respectively the numbers of healthy peeople, pre-diabetics and diabetics without complications, diabetics with complications and diabetics become disabled. We consider the model developed by Boutayeb et al [3]:

$$\begin{aligned}
\dot{H} &= \rho - \sigma_1 H - (\sigma_2 + \sigma_3 + \mu)H + \gamma_1 E \\
\dot{E} &= \sigma_1 H - (\gamma_1 + \mu + \beta_1 + \beta_3)E + \gamma_2 D \\
\dot{D} &= \sigma_2 H + \beta_1 E - (\mu + \beta_2 + \gamma_2 + \nu_2)D + \gamma_3 C \\
\dot{C} &= \sigma_3 H + \beta_3 E + \beta_2 D - (\mu + \delta + \gamma_3 + \nu_1)C \\
\dot{B} &= \nu_2 D + \nu_1 C - (\mu + \tau)B
\end{aligned} \tag{1}$$

where ρ is the incidence of healthy adult population. The rate of healthy persons to become diabetic without complication and diabetic with complication, is described by the parameters σ_2 and σ_3, respectively. Parameter μ is natural mortality rate. Parameter γ_1 and σ_1 are related to the rate at which a pre-diabetic person becomes healthy and vice versa. We denote γ_2 as the rate of a diabetic person to become pre-diabetic, and γ_3 as the rate at which a diabetic with complications become diabetic without complications. Parameter ν_1 is related to the rate at which a diabetic with complications become disabled and ν_2 is the rate at which a diabetic person become disabled. The probability of a pre-diabetic person to become diabetic is denoted by β_1. Parameter β_2 represents the probability of a diabetic person developing a complications. The probability of a pre-diabetic person developing a complication is denoted by β_3. By τ and δ we denote the mortality rate due to disabled and mortality rate due to complications, respectively. All parameters have positive value.

The controlled model is given by the following system

$$\begin{aligned}
\dot{H} &= \rho - \sigma_1 (1-u)H - (\sigma_2 + \sigma_3 + \mu)H + \gamma_1 E \\
\dot{E} &= \sigma_1 (1-u)H - (\gamma_1 + \mu + \beta_1 + \beta_3)E + \gamma_2 D \\
\dot{D} &= \sigma_2 H + \beta_1 E - (\mu + \beta_2 + \gamma_2 + \nu_2)D + \gamma_3 C \\
\dot{C} &= \sigma_3 H + \beta_3 E + \beta_2 D - (\mu + \delta + \gamma_3 + \nu_1)C \\
\dot{B} &= \nu_2 D + \nu_1 C - (\mu + \tau)B
\end{aligned} \tag{2}$$

where u is a control. The objective function is defined as

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$$J(u)=\int_0^{t_f}(E(t)+Au^2(t))dt \tag{3}$$

where $E(t)$ represents pre-diabetics who want to be minimized. The constant A is a positive weight that balances the size of u, $t=0$ is initial time, and t_f is final time. U is the control set defined by

$$U=\{u:0\le u\le 1, t\in[0,t_f]\}$$

The optimal control $u^*\in U$ satisfies $J(u^*)=\min_{u\in U} J(u)$.

3.2. Global Stability for Autonomous Linear System

In this section we prove stability of the system by considering the Lyapunov properties of the linear system.

Theorem 1 (Lyapunov's theorem) [5]. Consider $\dot{x}=f(x), x\in\mathbb{R}^n, f:\mathbb{R}^n\to\mathbb{R}^n$. Let $D\subset\mathbb{R}^n$ and let $x=0$ be contained in D. Let $V(x)$ be a continuously differentiable function such that $V(x)>0,\ \forall x\in D-\{0\}$ and $\dot{V}(x)\le 0,\ \forall x\in D$ then $x=0$ is stable. If, in addition, $\dot{V}(x)<0,\ \forall x\in D-\{0\}$ then $x=0$ is asymptotically stable.

Lyapunov's method requires one to choose a positive definite Lyapunov function (candidate) and then prove that its derivative is negative (semi) definite. Quadratic Lyapunov functions can be used to test stability of linear systems.

Theorem 2 (Quadratic form of Lyapunov function) [5]. Consider $\dot{x}=A(x), x\in\mathbb{R}^n$. The system (origin) is globally asymptotically stable if and only if there exists a positive definite matrix $P=P^T>0$ such that A^TP+PA is negative definite or $A^TP+PA<0$. Equivalently if, for a given $Q=Q^T>0$, it is possible to find a $P=P^T>0$ such that

$$A^TP+PA=-Q \tag{4}$$

then system is globally asymptotically stable

Theorem 3 [5]. For a given $Q=Q^T>0$ there exists a unique $P=P^T>0$ satisfying the Lyapunov equation
$A^TP+PA=-Q$ so that the system (1) is globally asimtotically stable.

Proof:

Let
$$A=\begin{bmatrix}-\eta_1 & \gamma_1 & 0 & 0 & 0\\ \sigma_1 & -\eta_2 & \gamma_2 & 0 & 0\\ \sigma_2 & \beta_1 & -\eta_3 & \gamma_3 & 0\\ \sigma_3 & \beta_3 & \beta_2 & -\eta_4 & 0\\ 0 & 0 & \nu_2 & \nu_1 & -\eta_5\end{bmatrix}, A^T=\begin{bmatrix}-\eta_1 & \sigma_1 & \sigma_2 & \sigma_3 & 0\\ \gamma_1 & -\eta_2 & \beta_1 & \beta_3 & 0\\ 0 & \gamma_2 & -\eta_3 & \beta_2 & \nu_2\\ 0 & 0 & \gamma_3 & -\eta_4 & \nu_1\\ 0 & 0 & 0 & 0 & -\eta_5\end{bmatrix}$$

where

$\eta_1=\sigma_1+\sigma_2+\sigma_3+\mu,\ \eta_2=\gamma_1+\mu+\beta_1+\beta_3,\ \eta_3=\mu+\beta_2+\gamma_2+\nu_2,\ \eta_4=\mu+\delta+\gamma_3+\nu_1,\ \eta_5=\mu+\tau$

We choose $Q=Q^T=I$ so from equation $A^TP+PA=-I$, we obtain

$$P=\begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} & p_{15} \\ p_{12} & p_{22} & p_{23} & p_{24} & p_{25} \\ p_{13} & p_{23} & p_{33} & p_{34} & p_{35} \\ p_{14} & p_{24} & p_{34} & p_{44} & p_{45} \\ p_{15} & p_{25} & p_{35} & p_{45} & p_{55} \end{bmatrix}$$

where

$$\begin{bmatrix} p_{11} \\ p_{12} \\ p_{13} \\ p_{14} \\ p_{15} \\ p_{22} \\ p_{23} \\ p_{24} \\ p_{25} \\ p_{33} \\ p_{34} \\ p_{35} \\ p_{44} \\ p_{45} \\ p_{55} \end{bmatrix} = \begin{bmatrix}
-2\eta_1 & 2\sigma_1 & 2\sigma_2 & 2\sigma_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\gamma_1 & (-\eta_1-\eta_2) & \beta_1 & \beta_3 & 0 & \sigma_1 & \sigma_2 & \sigma_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \gamma_2 & (-\eta_1-\eta_3) & \beta_2 & \nu_2 & 0 & \sigma_1 & 0 & 0 & \sigma_2 & \sigma_3 & 0 & 0 & 0 & 0 \\
0 & 0 & \gamma_3 & (-\eta_1-\eta_4) & \nu_1 & 0 & 0 & \sigma_1 & 0 & 0 & \sigma_2 & 0 & \sigma_3 & 0 & 0 \\
0 & 0 & 0 & 0 & (-\eta_1-\eta_5) & 0 & 0 & 0 & \sigma_1 & 0 & 0 & \sigma_2 & 0 & \sigma_3 & 0 \\
0 & 2\gamma_1 & 0 & 0 & 0 & -2\eta_2 & 2\beta_1 & 2\beta_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \gamma_1 & 0 & 0 & \gamma_2 & (-\eta_2-\eta_3) & \beta_2 & \nu_2 & \beta_1 & \beta_3 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \gamma_1 & 0 & 0 & \gamma_3 & (-\eta_2-\eta_4) & \nu_1 & 0 & \beta_1 & 0 & \beta_3 & 0 & 0 \\
0 & 0 & 0 & 0 & \gamma_1 & 0 & 0 & 0 & (-\eta_2-\eta_5) & 0 & 0 & \beta_1 & 0 & \beta_3 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 2\gamma_2 & 0 & 0 & -2\eta_3 & 2\beta_2 & 2\nu_2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \gamma_2 & 0 & \gamma_3 & (-\eta_3-\eta_4) & \nu_1 & \beta_2 & \nu_2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \gamma_2 & 0 & 0 & (-\eta_3-\eta_5) & 0 & \beta_2 & \nu_2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2\gamma_3 & 0 & -2\eta_4 & 2\nu_1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \gamma_3 & 0 & (-\eta_4-\eta_5) & \nu_1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2\eta_5
\end{bmatrix}^{-1} \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \\ 0 \\ 0 \\ 0 \\ -1 \\ 0 \\ 0 \\ -1 \\ 0 \\ -1 \end{bmatrix} \quad (5)$$

Inverse matrix in Equation (5) exists because it has full row rank for all positive value of parameters, so there exists a unique P . In order to be positive definite, Matrix P must satisfy

$$\Delta_1 = p_{11} > 0, \Delta_2 = \begin{vmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{vmatrix} > 0, \Delta_3 = \begin{vmatrix} p_{11} & p_{12} & p_{13} \\ p_{12} & p_{22} & p_{23} \\ p_{13} & p_{23} & p_{33} \end{vmatrix} > 0, \Delta_4 = \begin{vmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{12} & p_{22} & p_{23} & p_{24} \\ p_{13} & p_{23} & p_{33} & p_{34} \\ p_{14} & p_{24} & p_{34} & p_{44} \end{vmatrix} > 0, \Delta_5 = \det(P) > 0 \cdot$$

3.3. Controlability for Nonlinear Control System

Definition 1 [5]. Consider two vector fields $f(x)$ and $g(x)$ in $\Re^n$. Then the Lie bracket operation generates a new vector field

$$[f,g] = \frac{\partial g}{\partial x}.f - \frac{\partial f}{\partial x}.g$$

Also, higher order Lie brackets can be defined

$$[ad_f^1, g] = [f, g]$$

$$[ad_f^2, g] = [f, [f, g]]$$

$$\vdots$$

$$[ad_f^k, g] = [f, [\mathrm{ad}_f^{k-1}, g]]$$

Theorem 4 [5]. The system defined by

$$\dot{x} = f(x) + \sum_{i=1}^{m} g_i(x) u_i$$

is locally accessible about x_0 , if the accessibility distribution K spans n space, where n is the rank of x and K is defined by

$$K = \left[g_1, \ldots, g_m, [g_i, g_j], \ldots, [\mathrm{ad}_{g_i}^k, g_j], \ldots, [\mathrm{f}, g_i], \ldots [\mathrm{ad}_f^k, g_i] \right]$$

Theorem 5. The system (2) is locally accessible.
Proof: System (2) has form

$$\dot{x} = f(x) + g(x)u$$

where

$$\dot{x} = \begin{bmatrix} \dot{H} \\ \dot{E} \\ \dot{D} \\ \dot{C} \\ \dot{B} \end{bmatrix},\ f(x) = \begin{bmatrix} \rho - \eta_1 H + \gamma_1 E \\ \sigma_1 H - \eta_2 E + \gamma_2 D \\ \sigma_2 H + \beta_1 E - \eta_3 D + \gamma_3 C \\ \sigma_3 H + \beta_3 E + \beta_2 D - \eta_4 C \\ \nu_2 D + \nu_1 C - \eta_5 B \end{bmatrix},\ g(x) = \begin{bmatrix} \sigma_1 H \\ -\sigma_1 H \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

And $\eta_1 = \sigma_1 + \sigma_2 + \sigma_3 + \mu$, $\eta_2 = \gamma_1 + \mu + \beta_1 + \beta_3$, $\eta_3 = \mu + \beta_2 + \gamma_2 + \nu_2$, $\eta_4 = \mu + \delta + \gamma_3 + \nu_1$, $\eta_5 = \mu + \tau$

Furthermore,

$$\frac{\partial f}{\partial x} = \begin{bmatrix} -\eta_1 & \gamma_1 & 0 & 0 & 0 \\ \sigma_1 & -\eta_2 & \gamma_2 & 0 & 0 \\ \sigma_2 & \beta_1 & -\eta_3 & \gamma_3 & 0 \\ \sigma_3 & \beta_3 & \beta_2 & -\eta_4 & 0 \\ 0 & 0 & \nu_2 & \nu_1 & -\mu \end{bmatrix},\ \frac{\partial g}{\partial x} = \begin{bmatrix} \sigma_1 & 0 & 0 & 0 & 0 \\ -\sigma_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Controllability matrix of system (2) is

$$K = \left[g, \left[ad_f^1, g \right], \left[ad_f^2, g \right], \left[ad_f^3, g \right], \left[ad_f^4, g \right] \right] \quad (6)$$

If we calculate $\left[ad_f^1, g \right]$ to $\left[ad_f^4, g \right]$ and substitute and then substitute it to the matrix K, we get $\det K \neq 0$ and for $H, E, D, C, B > 0$ controllability matrix K has rank 5, so system (2) is locally accessible.

3.4. Numerical Example

In this section, we provide a numerical example to illustrate the above model. For the numerical illustration of the developed model, the values of various parameters in proper units taken from [1-2]:

$$\rho = 6\times10^6, \sigma_1 = 0.2, \sigma_2 = 0.3, \sigma_3 = 0.1, \mu = 0.2,$$
$$\gamma_1 = 0.08, \beta_1 = 0.5, \beta_3 = 0.5, \gamma_2 = 0.08, \beta_2 = 0.5,$$
$$\gamma_3 = 0.08, \delta = 0.05, \nu_2 = 0.05, \nu_1 = 0.05, \tau = 0$$

and the values of H, E, D, C, B use the values of variabels at initial time as follows:

$$H(0) = 1.3\times10^7, E(0) = 6.66\times10^6, D(0) = 1.02\times10^7, C(0) = 0.55\times10^7, B(0) = 1.1\times10^8$$

Example 1. We check the stability by solving Equation (5) with the above parameters and variabels, then we obtain

$$P = \begin{bmatrix} 0.8462520010 & 0.4758618692 & 0.4279712148 & 0.4089608763 & 0.3485316102 \\ 0.4758618692 & 0.7435502165 & 0.4972065825 & 0.4462793929 & 0.3721608719 \\ 0.4279712148 & 0.4972065825 & 0.8370414302 & 0.5153816022 & 0.4969378406 \\ 0.4089608763 & 0.4462793929 & 0.5153816022 & 0.9805384388 & 0.6490813522 \\ 0.3485316102 & 0.3721608719 & 0.4969378406 & 0.6490813522 & 2.5 \end{bmatrix}$$

The minor of matrix P are

$$\Delta_1 = 0.8462520010 > 0, \Delta_2 = \begin{vmatrix} 0.8462520010 & 0.4758618692 \\ 0.4758618692 & 0.7435502165 \end{vmatrix} = 0.40278634>0,$$

$$\Delta_3 = \begin{vmatrix} 0.8462520010 & 0.4758618692 & 0.4279712148 \\ 0.4758618692 & 0.7435502165 & 0.4972065825 \\ 0.4279712148 & 0.4972065825 & 0.8370414302 \end{vmatrix} = 0.1942723976>0,$$

$$\Delta_4 = \begin{vmatrix} 0.8462520010 & 0.4758618692 & 0.4279712148 & 0.4089608763 \\ 0.4758618692 & 0.7435502165 & 0.4972065825 & 0.4462793929 \\ 0.4279712148 & 0.4972065825 & 0.8370414302 & 0.5153816022 \\ 0.4089608763 & 0.4462793929 & 0.5153816022 & 0.9805384388 \end{vmatrix} = 0.1181550861>0,$$

$$\Delta_5 = \det(P) = 0.239481916>0$$

Because the minor $\Delta_k > 0, k = 1,...,5$, it implies that P is positive definite matrix. System (1) is globally asymptotically stable.

Example 2. We check the controllability by solving Equation (6) with the above parameters and variabels, then we obtain

$$K = \begin{bmatrix} 240 & 372 & 602 & 579.3 & 679.15 \\ -240 & -420 & -278.8 & -453.3 & -964.85 \\ 0 & 24 & 105.6 & 18.72 & 127.656 \\ 0 & 24 & 67.2 & -76.92 & -268.598 \\ 0 & 0 & -7.2 & -25.8 & -17.418 \end{bmatrix} \quad (7)$$

Matrix K in Equation (7) has rank 5, so the system is locally accessible.

4. Conclusion

Based on the description and explanation of the chapter of results and discussion can be concluded that the problem of autonomous linear system stability can be solved by using Lyapunov method, by determining quadratic forms of Lyapunov function in system (1). By choosing identity matrix, we can find a positive definite matrix, so that resulting system (1) is globally asymptotically stable. For controlled models, controllability matrix is obtained by using Lie brackets without linierization of non-linear systems. Controllability matrix of system (2) has full row rank, so system (2) is locally accessible.

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