Multi agent through serret-frenet system

by R. Heru Tjahjana

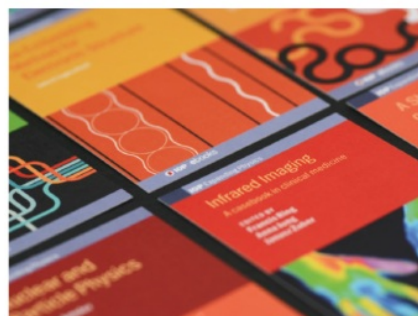**PAPER · OPEN ACCESS**

Multi agent through serret-frenet system

To cite this article: R H Tjahjana and R H S Utomo 2019 *J. Phys.: Conf. Ser.* **1321** 022071

View the article online for updates and enhancements.

Multi agent through serret-frenet system

R H Tjahjana* and R H S Utomo

Department of Mathematics, Faculty of Science and Mathematics, Diponegoro University, Semarang, Indonesia

*Corresponding author: heru.math.undip@gmail.com

Abstract. Multi agent system present in our daily live. The Multi agent system, starting from a few agents to tens and even millions of agents. Control of multi-agent is one of the most important problem in control theory. Many methods are used to control of multi agent, one of the method is optimal control. Approach with optimum control too many kinds. In this paper, the authors use the Serret-Frenet system. In mathematical models of multi-agent systems written, multi-Serret-Frenet dynamics systems are used. Some agents move with the initial formation from the starting position, towards the final position that is popularized. The movement of maintaining the formation and minimizing the cost of control is translated into the functional cost model. From the dynamic system model and functional model of cost is designed multi-agent control system with the optimum control approach.

1. Introduction

Multi agents appear naturally and artificially. In nature appears in animals that cluster, on water, land and air [1]. Examples in water or in the sea the fish clustered into a multi agent system with various reasons, for example for the safety of natural predators, or for increasing swimming speed or for reasons to scare their natural enemies [2]. In the air, the birds form a multi-agent natural system with a variety of interests, such as changing locations due to the influence of the season. By flying together as a multi agent, the flight range of birds flies even further [3]. On land, animals chase prey with a multi agent system with a view to increasing the chance of getting prey [4].

Besides appearing naturally, multi agent systems also appear as man-made systems. Man-made systems also appear on land, sea and in the air. A variety of modern equipment inspired by the multi agent system and several multi-agent system applications, appears in [5-20].

The human desire to control the multi agent system is carried out through sharing types of approaches. One approach used by humans is through a control system. Many approaches with the control system, in this paper the approach used is optimum control. In the optimum control approach a mathematical model is needed. The mathematical model for multi agents also has many kinds. In this paper, in addition to using mathematical models for multi agent models, there is one more mathematical model for modeling the collective work of agents, namely the functional objective model. The multi agent model used this paper is the Serret-Frenet model. The novelties of this paper are listed as follows: (1) the use of the non linear Serret-Frenet system as a multi agent system for 5 agents, (2) the optimal control approach with certain initial and final requirements (3) special cost functional that model the behavior of agents moving in formation from beginning to end, also each agent must not collide with each other at the same time, each agent cannot avoid each other. The next section discusses the mathematical model.

2. Mathematical Model

Specifically in this paper, clustering phenomena are modeled as multi-agent systems consisting of several independent dynamic systems, but with one functional depending on each state variable in each dynamic system. With the initial and final state formation given, control must be found, so that each agent is in the formation that is determined at the beginning and is as well as desired until the end, by minimizing J. More precisely, we must minimize objectives functional or cost functional

$$J = \int_0^T \sum_{k \neq j} \frac{\mu}{\left\| r_k - r_j \right\|^2} + \tfrac{1}{2}(u_1^2 + \ldots + u_k^2) dt \tag{1}$$

with r_k and u_k meet the Serret-Frenet system as follows

$$\dot{r}_k = x_k$$
$$\dot{x}_k = y_k u_k \tag{2}$$
$$\dot{y}_k = -x_k u_k$$

where $[x_k, y_k]$ is a moving orthonormal skeleton and u_k is control. Whereas r_k the kth −agent position vector in R^2. Initial condition $x_k(0), y_k(0), r_k(0)$ and final condition $x_k(T), y_k(T), r_k(T)$ are given designate the initial and final position and orientation that must be met by the agents.

In the next theorems will discuss about convexity of cost functional J and existence of optimal control. These theorems are important before determination of optimal control.

Theorem 2.1

Cost Functional J which describe in (1) is convex

Proof:

Consider form of cost functional are summations of the fraction with the form of the denumerator is quadratic and the other one are summation of quadratic form. The quadratic form is convex, so the fractions with the denumerators are convex then the fractions forms are convex too. Also the summations of quadratic forms are convex function too. The conclusion is cost functional J is convex. ∎

The next discussion is about the existence of optimal control. The existence of optimal control u_k in equation system (2) and minimization cost functional (1) exposed in the next Theorem.

Theorem 2.2

Optimal control u_i for $i = 1,2,\ldots,k$ in equation system (2) and minimization cost functional (1) are exist

Proof:

Onsider Theorem 2.1, cost functional (1) is convex, so the existence control u_i for $i = 1,2,\ldots,k$, are guaranteed exist. ∎

Consider cost functional or objectives functional (1). This objective functional equation consists of two parts. The first part is used to force agents not to collide with each other. The second part is the cost of controlling the agents. These two tribes must be minimized. Next, final condition $x_k(T), y_k(T), r_k(T)$ do not enter into functional costs (1). This is what distinguishes this paper from other papers that use the Serret-Frenet approach and optimum control.

Next, the process of determining the control for each agent uses the Potryagin Minimum Principles (PMP) approach as follows. From equation (1) and equation (2) the Hamilton function is formed. From the Hamilton function, the Hamiltonian System can then be derived. With PMP, it is necessary to obtain the necessary conditions from the controllers u_k. The equation for these controls is then substituted into

the hamiltonian system and together the initial and final conditions are solved numerically then the simulation results are given in the following section.

3. Simulation of Serret-Frenet Multi Agent Model
In the multi agent control simulation on this paper, it is done for five agents. Each agent can be assumed as a dot model and each agent also fulfills the dynamic system presented in (2). Agents are required to fulfill the initial and final conditions as the initial and final position for each agent given in the Table 1 as follows.

Table 1. Initial and Final Position Each Agent

Agents	Initial Positions	Final Positions
Agent 1	(2,2.73205808)	(22,22.73205808)
Agent 2	(1.5,1.866025)	(21.5,21.866025)
Agent 3	(2.5,1.86602540)	(22.5,21.86602540)
Agent 4	(1,1)	(21,21)
Agent 5	(3,1)	(23,21)

Simulation scenarios are given as follows. At the initial position the agent forms a certain formation, which is a triangle. During the time between the start and end times the moving agents must not collide with each other. This simulation scenario between start and end times is also accommodated in objective functional (2). At the end of the time given also returns to form a triangle.

In addition to the initial and final requirements, the agents are also related to each other in the functional objective (1). The success of the simulation results is given in Figure 1. In Figure 1, the agents have succeeded in moving from the initial position to the final position within the allotted time.
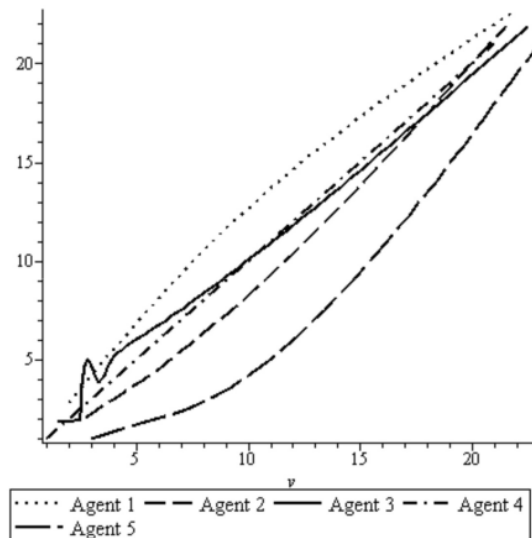

Figure 1. Optimal path of five agents

Consider Figure 1, the movement of the agents start from initial position with triangle formation until final position with the same formation. Through optimal control approach, the control of each agent is obtained and from these contol values, optimal paths which ploted in Figure 1 are obtained. With the cost functional given in (1) and the multi agent system (2) with the optimal control approach for 5 agents

a differential equation system will be obtained with 30 equations with incomplete initial and final requirements. The simulation results show that in a given time, that is, from the beginning to a certain end time, the agents can move in one herd and not collide with one another.

Implication of this study in world applications give us some ideas on how this research will be utilized in industries as long objective the research. This study has potency to produce control design of multi-agent that moves in formation. The control design is utilize in telecommunication, object searching and art performance.

4. Conclusion

As a conclusion, the multi agent model using Serret-Frenet for five agents is successfully controlled with optimum control. Agents that are required to move do not collide from the initial position to the final orientation. The optimum control approach successfully completes the control simulation of the agents. The agents are modeled to move along the Serret-Frenet dynamics system. The agents are related to each other in an objective functional. As an important result, the optimal path of each agent are plotted in Figure 1.

Acknowledgment

References

[1]   Liu Y and Passino K 2000 *Swarm Intelligence: Literature Overview* (Neil Ave, Columbus: The Ohio State University)
[2]   Huse G, Railsback S and Ferno A 2002 *J. Fish Biol* **60** 571
[3]   Seiler P, Pant A and Hedrick K 2003 *J. Theory Biol* **59221** 279
[4]   Weisel I 2006 *Predatory and Foraging Behaviour of Brown Hyenas at cape fur Seal colonies*, Ph.D. Thesis (Hamburg: University of Hamburg)
[5]   Fink J, Michael N, Kim S and Kumar V 2011 *Int. J. Robotic Res* **30** (3) 324.
[6]   Duan H, Luo Q and Yu Y 2013 *SCI. CHINA Technol. Sci* **56** 1066
[7]   Duan H, Luo Q, Shi Y and Ma G 2013 *IEEE Comput. Intell. Mag* **8** 16
[8]   Ortiz A, Derek K and Cédric L 2013 *J. Intell. Robotic Syst* **70** 255
[9]   Perez D, Ivan M, Fernando C, David S, Enrique C and Anibal O 2013 *J. Intell. Robotic Syst* **69** 119
[10] Turpin M, Michael N and Kumar V 2012 *Auton. Robots.* **33** 143
[11] Vásárhelyi G, Virágh C, Somorjai G, Tarcai N, Szörényi T, Nepusz T and Vicsek T 2014 *Proc. Of IEEE IROS 2014 Conference.*
[12] Won D Y, Göktoğan A H, Sukkarieh S and Tahk M J 2013 *Front. Intell. Auton. Syst. Stud. Comput. Intell.* **466** 193
[13] Yuan Y, Yan JG and Qu YH 2013 *Flight dyn.* **31** 29
[14] *Shakshuk* E *and Reid* M 2015 Procedia Comput. Sci. **52** 252
[15] Xie J and Liu C 2017 *J. Int. Counc. Electr. Eng* **7** 188
[16] Li F and Xu Z 2018 *PLoS ONE* **13** e0205445
[17] Nguyen T L, Guillo-Sansano E, Syed M H, Nguyen V H, Blair S M, Reguera L, Tran Q T, Caire R, Burt G M, Gavriluta C and Luu N A 2018 *Energies* **11** 3253
[18] Hu W, Liu L and Feng G 2016 *IEEE Trans. Cybern* **46** 148
[19] Zuo Z and Tie L 2016 *Int. J. Syst. Sci.* **47** 1366
[20] He W, Chen G, Han Q and Qian F 2017 *Inf. Sci* **380** 145

Multi agent through serret-frenet system