# Optimal Path Determination for Flying Vehicle to Search an Object

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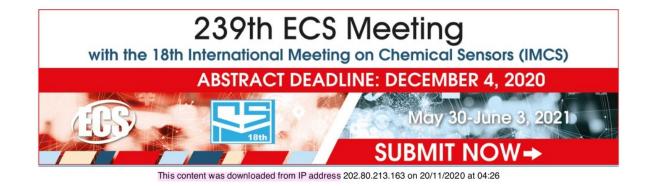
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# **Optimal Path Determination for Flying Vehicle to Search an** Object

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Abstract. In this paper, a method to determine optimal path for flying vehicle to search an object is proposed. Background of the paper is controlling air vehicle to search an object. Optimal path determination is one of the most popular problem in optimization. This paper describe model of control design for a flying vehicle to search an object, and focus on the optimal path that used to search an object. In this paper, optimal control model is used to control flying vehicle to make the vehicle move in optimal path. If the vehicle move in optimal path, then the path to reach the searched object also optimal. The cost Functional is one of the most important things in optimal control design, in this paper the cost functional make the air vehicle can move as soon as possible to reach the object. The axis reference of flying vehicle uses N-E-D (North-East-Down) coordinate system. The result of this paper are the theorems which say that the cost functional make the control optimal and make the vehicle move in optimal path are proved analytically. The other result of this paper also shows the cost functional which used is convex. The convexity of the cost functional is use for guarantee the existence of optimal control. This paper also expose some simulations to show an optimal path for flying vehicle to search an object. The optimization method which used to find the optimal control and optimal path vehicle in this paper is Pontryagin Minimum Principle.

### 1. Introduction

The researchs about the flying vehicle attract the attention of the researchers. Many researches has conducted the research about flying vehicle. The examples of recent research related with the flying vehicle topic are [1]-[8], these papers are different from this this paper, these papers do not use optimal control approach. The results of the researchs which related with the flying vehicle research conducted by the main authors expose about fuel saving can be read in [9]-[10]. This paper describe model of one flying vehicle that move to search an object use optimal control approach. The exposition of this paper inspired by sad event like an aircraft fall or miss in the midle of its flight. After fall or after performs emergency landed prosess in the land or in the water an aircraft produce special signal. This special signal use to help the search and rescue team to find the loss aircraft. The searcher aircraft or the other aircraft will catch the signal from the loss aircraft and the seracher aircraft will move reach it. The detail of the main problem of this paper will expose in the next section.

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# 2. Problem Statement

Searching an object in wide area is an interesting problem. This exposition describe the object with special signal that loss in the field and the flying vehicle has a task to find it. This paper will expose how to find the object with the flying vehicle. The problem of this paper is how to determine optimal path which describe a flying vehicles to search an object. The flying vehicle after catch the signal from the object then fly to the object. The main problem is how to make the vehicle flying along the optimal path and as soon as possible reach the lost object.

# 3. Mathematical Model

This paper use optimal control to solve the problem. In optimal control approach, the mathematical model is needed. The assumption is need to expose before the model is chosen.

# 3.1 Assumption

Beside the object which searched completed by a signal producer during searching time, this paper make some assumptions as follows: the flying vehicle move in the same height or mathematically move in two dimentions, no disturbance between object and flying vehicle, and the vehicle has fuel enough along the motion to reach the object.

# 3.2 The model of flying vehicle

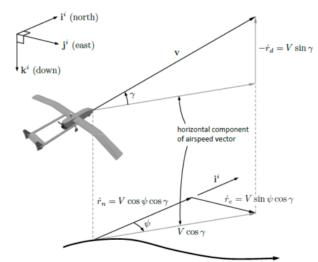
The model in this paper divides into two parts. The first model is model of the flying vehicle and the second one is model for describe the vehicle's task. In this paper the model of flying vehicle use Dubin's model as follows

$$\dot{r_n} = V \cos \psi \cos \gamma^c$$

$$\dot{r_e} = V \sin \psi \cos \gamma^c$$

$$\dot{r_d} = V \sin \gamma$$

$$\dot{\psi} = \frac{g}{v} \tan \phi^c$$
(1)



Flight path projected onto ground

Figure 1. Explanation of the state variables [11]

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Under assumption that the vehicle move in the same height along the motion then model in equation(1) can be simplified as follows

$$\dot{r}_{n} = u_{1} \vee \cos \psi$$

$$\dot{r}_{e} = u_{1} \vee \sin \psi$$

$$\dot{r}_{d} = 0$$

$$\dot{\psi} = u_{2}.$$
model (2) can make simpler as follows
$$\dot{r}_{n} = u_{1} \cos \psi$$

$$\dot{r}_{i} = u_{i} \sin \psi$$
(2)
(3)

In this paper, 1

$$\begin{aligned} r_n &= u_1 \, \cos \psi \\ \dot{r}_e &= u_1 \, \sin \psi \\ \dot{\psi} &= u_2. \end{aligned} \tag{3}$$

#### 3.3 Model of Cost Functional

Cost functional is the most important thing in optimal control. The cost functional or index performance make the system works satisfy the goal to solve the problem. Since the goal of the paper writing is how can to take position an serched object as soon as possible then we use cost funtional that must be minimized as follows

$$J = \int_0^T dt. \tag{4}$$

### 4. Mathematical Analysis

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#### 4.1 Controllability and Observability

Since the solution of this paper through optimal control, the mathematical analysis is needed. The first Analysis is Controllability. The controllability model (3) is needed to checked.

#### Theorem 4.1.1

The model of vflying vehicle is controllable

Proof:

If model (3) linearized, the result is not controllable. The controllability can show via non linear system. The process to show that the model controllable can start from to write the model to Affine form. The Affine form of the model (3) can be presented as follows

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + g_1 u_1 + g_2 u_{2,} \tag{5}$$

with 
$$\mathbf{x} = \begin{bmatrix} r_{\mathbf{n}} \\ r_{\mathbf{e}} \\ \psi \end{bmatrix} \mathbf{f}(\mathbf{x}) = \mathbf{0}, \mathbf{g}_{1} = \begin{bmatrix} \sin r_{\mathbf{n}} \\ \cos \psi \\ 0 \end{bmatrix}$$
 and  $\mathbf{g}_{2} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ . Then the process is determination of

 $\Gamma_{cin} W_1$ 

controllability matrix. In this process, need Lie Bracket is needed. Finally, the controllability matrix has rank 3. From this fact, can be concluded that the system is locally accessible and since f(x) = 0then the system is controllable.

The next analysis is observability. The observability of the model given in the next theorem as follows.

# Theorem 4.1.2

The model of vflying vehicle is observable

Proof:

To check the observability, the observability matrix is formed. After that, the rank of observability matrix is computed. In this case, the observability matrix is 3, and can be conclude that the model (3) is observable.

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# 4.2 Existence of Optimal Control

Optimal control is used in this paper, so the existence of optimal control is needed to showed. The existence theorem is given as follows.

## Theorem 4.2.1

The optimal control for the peoblem of this paper is exist Proof:

The convexity of the cost functional is guarantee that the optimal control is exist. Consider the cost functional which given in equation number (4). The integrad is constant function. The constant function is convex, so the cost fountional is convex. If the cost functional is convex then the optimal control is exist. The other things that guarantee the existence of optimal control is restriction of the control. Consider from the declaration of the Dubin's aircraft model that the controls are bounded, from these facts can be conclude that the existence of optimal control is guaranteed.  $\blacksquare$ 

### 4.3 Optimal Path

After the existence of optimal control is guareanted, if the optimal control is used for drive the flying vehicles from the origin to reach the searched object then the vehicle's path that obtained is optimal path. The path that produced with the optimal control is optimal path. The optimal path also can be seen from the value of Hamiltonian Function. Hamiltonian function is connect between model of flying vehicle or dynamical system and cost functional. The value of Hamiltonian function is constant as long as in optimal path. In other words, the derivative of Hamiltonian function with respect to time as long as optimal trajectory is zero.

# 4.4 Method to determination optimal path

For determination optimal path this paper use optimal control approach. The optimal path begin from the initial condition of the searcher flying vehicle until the position of searched object. The proposed method to determine optimal path can be seen as optimal control problem as follows:

- 4.4.1 The dynamical system use the dynamical model of flying vehicle.
- 4.4.2 The initial condition use initial position and orientation the searcher vehicle and the boundary condition use the initial position and orientation from the serched object which detected from the signal that produced by the searched object.
- 4.4.3 The task of the flying searcher vehicle is move as soon as possible to reach the searche object. This task described in cost functional (4). The existence of optimal control is guaranteed from section 4.4.4. From the Hamiltonial function can be derivated into Hamiltonian system.
- 4.4.4 The optimal path equation is the solution of differential equation system which contain Hamiltonian system, optimal control, initial and boundary condition.

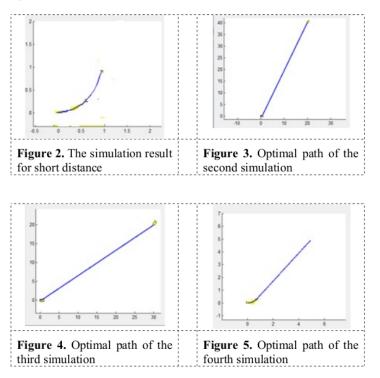
# 5. Simulation

#### 5.1 Simulation Scenario

The simulation begin from the lost object in the wide field. Like in the assumption the lost object produce special signal. The signal can detected by a flying vehicles, as a position that must be reached by the flying vehicles as soon as possible. The initial position of the flying vehicle and the position that must be reached or position of searced object can be seen as Two Points Boundary Value Problem. If the initial position and orientation of flying vehicle is given as  $X_0$  and the position and orientation of flying vehicle, forced fly from the initial position to the the reached object with minimized the time. This simulation scenario approaching the real situation when the object lost in wide area and as long as in searching time the object produce special signal.

#### 5.2 Simulation Result

The first simulation is to show that optimal control can ce used to solve the main problem in relative short distance and the result given in the Figure 2. The unit in this simulation sectionas follows (km, km, radian). The axis in simulation result is described as the erect and the horizontal axis. The erect axis is North and the horizontal axis is East. In the first simulation the initial condition of flying vehicle is (0,0,1) and the position of the searched object is (1,1,1). The initial conditions of the second, the third and the fourth simulations are as same as the first simulation (0,0,1) but the bondary condition of the second, the third and the fourth simulation result respectively are given by (20,40,1),(30,20,1) and (5,5,1) also the simulation result respectively are given in Figure 3, Figure 4 and Figure 5. From these figures can be maked conclusion that optimal control approach can work well to solve the problem.



### 6. Conclusion

Problem to search an object and during the searching time the lost object produce special signal which catched by the flying vehicle as a searcher vehicles can be solved by optimal control approach. From the mathematical analysis, the model (3) is feasible to solve the problem through optimal control method. The flying vehicle forced as soon as to reach the lost object by the cost functional. From the simulation results obtained conclusion that optimal control can be used to solve the main problem.

## 7. References

 Faessler M, Fontana F, Forster C, Mueggler E, Pizzoli M and Scaramuzza D 2016 Journal of Field Robotic 33 431 4th International Conference on Operational Research (InteriOR)

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IOP Conf. Series: Materials Science and Engineering 300 (2018) 012041 doi:10.1088/1757-899X/300/1/012041

- [2] Huang A S, Bachrach A, Henry P, Krainin M, Maturana D, Fox D and Roy N 2017 Springer Tracts in Advanced Robotics 100 235
- [3] Bhandari, S, Brown, Nc, Guzman, I, Sherman, T, Tellez J and Gomez E 2017 *Proc. AIAA SciTech Forum* AIAA 2017-1153
- [4] Zhang B and Duan H 2017 IEEE/ACM Transactions on Computational Biology and Bioinformatics 14 97
- [5] Conrad J, Wallach D, Kalweit F, Lindel P and Templin S 2017 Lecture Notes in Computer Science 10272 424
- [6] Siddall R, Ancel OA and Kovač M 2017 Journal of the Royal Society Interface Focus 7, 20160085
- [7] Andert F, Ammann N, Krause S, Lorenz S, Bratanov, D and Mejias L 2017 Journal of Intelligent & Robotic Systems 87 1
- [8] Aguilar W G, Luna M A, Moya J F, Abad V, Parra H and Ruiz H 2017 Proc. Int. Conf. on Semantic Computing (ICSC) San Diego CA p 509
- [9] Tjahjana, R H and Sasongko P S 2014 Applied Mathematical Sciences 8 733
- [10] Tjahjana, R H and Sasongko P S 2014 Applied Mathematical Sciences 8 5689
- [11] Owen M, Beard RW and Mc Lain T W 2015 Handbook of Unmanned Aerial Vehicles ed. Valavanis, KP and Vachtsevanos, GJ (Dordrecht: Springer) p 1677

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