

# Serret-Frenet Multi-Agent System with Optimal Control Approach

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# Serret-Frenet Multi-Agent System with Optimal Control Approach

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**Abstract.** Multi-agents systems is a system consisting of several agents, from two to hundreds agents. These agents are connected to each other. The main problem is how to control such systems. This paper inspired from some facts as follows. In nature, can be viewed animals perform multi agent system in the land and in the air. In avionic world, can be observed that there are many cases where pilots must fly their aircrafts in some particular formation. Some ships also very often form a convoy. In smaller scales, can be viewed that there are many robot designer would like to create multi-robot system for conducting some task. In particular, this paper focuses on the robotics utilizations. That is we would like to steer the multi-agent system to go from one particular position to the other position. Moreover, the agents move in some formations is required. The first objective of this paper is to create mathematical model of multi-agent system applicable in robotic. The model would consist of Serret-Frenet control systems. The second objective is to model the performance index of the multi-agent system. Thus, we would like the multi-agent system move from one place to another and the whole group of agents works collaboratively for optimizing the performance index. This is the idea behind multi-agent control system design. The third objective is to analyze controllability the above models. The last objective is to find the optimal control of each agent. The Pontryagin Maximum Principle is used to search this optimal control. The result of this paper, optimal control can be used to model the multi agent system Serret-Frenet system.

## 1. Introduction

The phenomenon of multi agent systems often occurs in nature and it is often revealed by animals [1-4]. Animal behavior that naturally forms a multi agent system, ranging from two to hundreds or even thousands of agents is an example of the optimality that nature can provide. Seeing the symptoms of optimality that occur in nature and as a form of admiration for nature, humans mimic the behavior of animals that form a multi agent system in nature. By mimicking the phenomena that exist in nature means the multi agent system not only occurs in animals in nature, but also appears in human-made phenomena. The importance of this research can be explained as follows. The use of multi agent systems in fields created by humans plays an important role in the era of industrialization. The use of multi agent systems in various fields demands studies related to multi agent control [5-10]. Viewed from the multi agent side, the problem of determining the control of multi agent systems is also interesting to study because it involves several control disciplines such as nonlinear controls, optimal controls and control problems with initial conditions and boundary conditions. For information, research on multi agents can also not involve control theory. Multi agent discussion that does not involve controls for example can be seen in the reference [11]. The research in multi agent which

involve control theory can be seen in [6-10]. Next, this paper discusses the mathematical model used in this paper.

## 2. The Model

In this paper, the mathematical model used consists of two parts. The first part is a system dynamics model for agents. The first part model uses the Serret-Frenet model provided by reference [12]. The selection of this model argues that the Serret-Frenet model is a relatively wellknown nonlinear model. In another article the author explores Serret-Frenet for 5 agents [13]. What if the Serret-Frenet model is used as a dynamic model in a multi agent system that is the motivation for the writer to present in the paper about multi agents with the Serret-Frenet approach. The second part is a model that describes the tasks with agents, namely moving from the initial position to the final orientation in formation and not colliding with each other. This second part model is presented in the form of a functional cost model. The initial position and orientation are used as initial conditions, while the final position and orientation are used as the final condition. This final position and orientation is not included in the second part model or the functional cost model.

For information, in more detail the first part of the mathematical model or the multi agent dynamics system model used in this paper, is given as follows.

$$\begin{aligned}\dot{r}_k &= x_k \\ \dot{x}_k &= y_k u_k \\ \dot{y}_k &= -x_k u_k\end{aligned}\quad (1)$$

where  $[x_k, y_k]$  is a moving orthonormal skeleton and  $u_k$  is control. Whereas  $r_k$  the  $k^{th}$  -agent position vector in  $R^2$ . The state variables of this model are  $r_k, x_k$  and  $y_k$ . While the second part model or cost functional model is presented as follows

$$J = \int_{t_0}^{t_f} \frac{\mu}{\|r_k - r_j\|^2} + \gamma \|r_k - r_j\|^2 + \sum_{k=1}^n u_k dt. \quad (2)$$

Consider equation (2),  $\mu$  is constant which make the first term forces each agent does not collide one to the other. Still in (2),  $\gamma$  is constant which make the second term forces each agent does not move far each other. From the mathematical model and functional cost model, the dynamic system model or Serret-Frenet model is controllable. Controllability of the system (1), can be seen in the reference [12]. Next, because this paper uses the optimum control approach, the theorem that guarantees optimum existence control needs to be given. A further theorem that guarantees the existence of optimum control is guaranteed by showing that functional costs are a convex function. Because functional costs are the sum of each tribe, each of which is a convex function, the functional cost is expressed as a convex function. Formally, the optimum existence of controls used as the main method in this paper is presented in the following theorem.

### Theorem 2.1.

*Optimal control for  $u_k$  guaranteed exist.*

Proof:

Take attention to equation (2). Equation two consists of three tribes. The first term is the division between a parameter and the quadratic function, because the quadratic function is a convex function so the first term is a convex function. The second term is the multiplication of a parameter with the quadratic function. Because the quadratic function is a convex function, the second term is also a convex function. Furthermore, the third term is a finite sum of each convex tribe, so the third term is a convex function. Because all the terms are convex functions, we conclude that the integrand equation (2) is convex and consequently the optimum control for  $u_k$  are guaranteed . ■

As an analysis of Theorem 2.1, without this theorem the result of this paper can not be guaranteed. Existence of each control plays important rule in optimal control determination. It is impossible to determine the optimal control, if the controls donot exist. The theorem 2.1 guarantees the existence of each control. After discussing the model, the next section discusses simulation.

### 3. The Simulation

Before the simulation results are given, first the simulation scenario is given in this paper. Simulation scenario in the initial time, the agents are in the initial position and orientation with a triangle formation. Furthermore, the agents move with the formation in the final position and orientation that has been determined by still being informed. Similarly, the travel time from the beginning to the end of the simulation is given before the simulation is done. The simulation scenario in this paper is done for three agents which in the initial and final formations form triangular formations. To obtain the simulation results first, note the multi agent model in equation (1) and functional costs (2). After forming the Hamilton function, and the Hamiltonian system and looking for the necessary conditions through Pontryagin Maximum Principle for achieving optimality, the control equation for each agent is obtained. After the control equation for each agent is substituted to model (1) again, the optimal path of the agents can be obtained and the simulation results are given in Figure 1. In Figure 1 it appears that according to the simulation scenario the agents move from the initial position and orientation to position goals required as final conditions. For different  $\mu$  and  $\gamma$  parameter values, results may not be exactly the same as Figure 1. This can happen, because the settlement uses a numerical approach. Substituted parameter values will greatly affect the results presented in Figure 1. In Figure 1, the agent 1 and 2 trajectories are as if they appear to intersect, so it seems as if these two agents are speaking. Even if seen from the time it arrived at the intersection, the two agents did not meet at the same time, so agent 1 and agent 2 never collided.

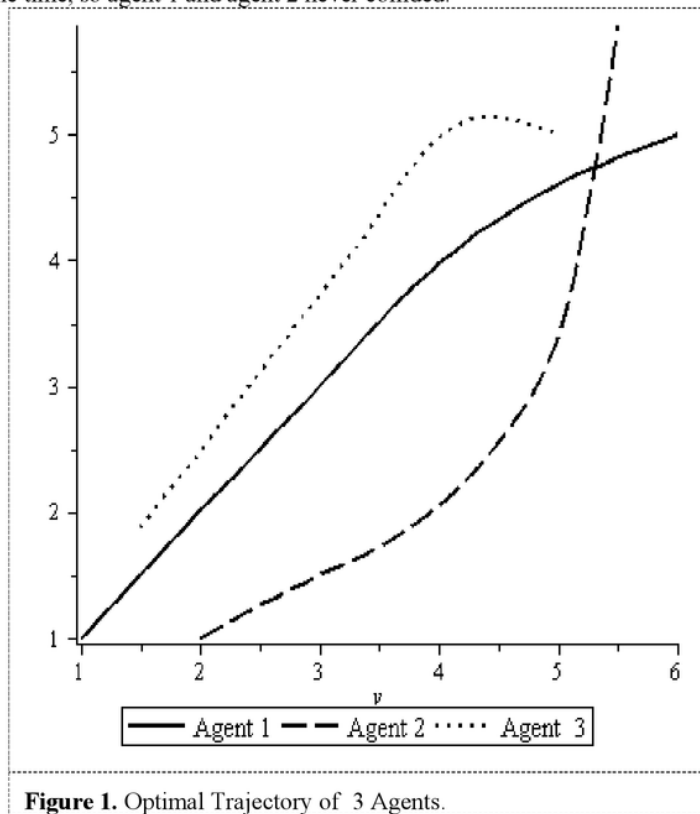


Figure 1. Optimal Trajectory of 3 Agents.

As a comparison with previous study which expose in introduction, the previous studies or papers do not use Serret-Frenet model together with optimal control in multi agent modeling.

#### 4. Conclusion

From the problem of controlling nonlinear multi agents with the Serret-Frenet model exposed in this paper it was concluded that the optimal control approach can be used to solve problems properly. From the simulation results made, the simulation scenario was successfully fulfilled. In the simulation results that have been done the agents can be forced to move in formation. The initial formation is a triangle, as well as the final formation of the agents still forming a triangle. In the simulation result, can be seen that optimal control can be used in multi agent modeling through Serret-Frenet model. As well as the agents can move by not colliding with each other. For different parameter values it may be possible to give results that are not exactly the same as in simulation result, but more or less the optimal trajectory plot for agents will be similar to the plot of simulation result.

#### 5. References

- [1] Liu Y and Passino K 2000 *Swarm Intelligence: Literature Overview* (Neil Ave, Columbus: The Ohio State University)
- [2] Huse G, Railsback S and Ferno A 2002 *Journal of Fish Biology* **60** 571
- [3] Seiler P, Pant, A and Hedrick, K 2003 *J. Theory Biology* **59221** 279
- [4] Weisel I 2006 *Predatory and Foraging Behaviour of Brown Hyenas at cape fur Seal colonies*, Ph.D. Thesis (Hamburg: University of Hamburg)
- [5] Ahmadzadeh, A, Motee, N, Jadbabaie, A and Pappas, G. 2009 *Proceeding of IEEE International Conference on Robotics and Automation 2009* 2449
- [6] Arrichiello F, Chiaverini S and Fossen, TI 2006 *Lecture Notes in Control and Information Systems series*, **336**
- [7] Breivik M, Hovstein VE, Fossen, TI 2008 *Proceeding of IFAC World Congress Seoul*.
- [8] Bouabdallah, S. 2007 *Design and Control of Vehicles with Application to Autonomous Flying*, PhD Thesis, (Lausanne : Ecole Polytechnique)
- [9] Borhaug E, Pavlov A and Pettersen, KY 2006 *Proc. 45th IEEE Conference on Decision and Control*, (San Diego: CA, USA)
- [10] Fink J, Michael N, Kim S, and Kumar V 2011 *The International Journal of Robotics Research*, **30** 3, 324.
- [11] Vidal J 2010 *Fundamentals of Multiagent System*, University of South Carolina
- [12] Justh, EW and Krishnaprasad, PS 2004 *Systems and Control Letters* 52 : 25–38.
- [13] Tjahjana, RH and Utomo, RHS 2018, Multi Agent Through Serret-Frenet System, Preprint

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