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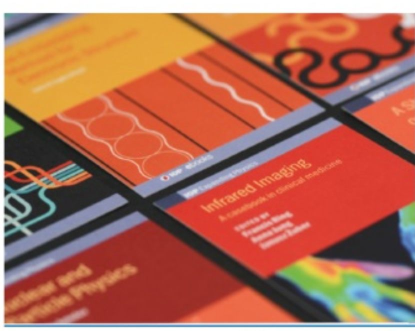

Some methods for identifying redundant constraints in linear programming

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Abstract. Linear programming problems consist of two parts are objective function and inequality constraints. A redundant constraint is a constraint that does not change the feasible region. There are many methods for detecting redundant constraint. The methods for identifying redundant constraints which in the process only uses the objective function and inequality constraints, among others, Heuristic method, Llewellyn method, and Stojkovic-Stanimirovic methods. Heuristic method cannot identify weakly redundant constraints as redundant constraints. Llewellyn method comparing two constraints. Stojkovic-Stanimirovic depend on objective function. In this paper, we give some examples to recognize characteristics of these methods.

1. Introducing

Linear programming represents a mathematical model for solving optimal allocation of resource [1]. Large-scale linear programming problems are many constraints and variable decisions [2]. The objective function and constraints are described by mathematical expressions [2]. Optimization is the act of obtaining the best result. The aim of all such decisions is to maximize (minimize) the function objective [3]. Simplex method and interior point method are main methods for solving linear programming problems. Determine initial point is a step necessary for these two methods [4]. Method Simplex is the most popularly known algorithm for LP [5].

Large-scale LP problems contain a considerable number of redundant constraints [6]. Redundancy may even have some unfavorable effects [7]. The importance of detecting and removing redundancy is the avoidance consume extra computational effort when solving an associated LPP [8, 9]. Duplicate rows are rows that are identical up to a scalar multiplier [10]. Duplicate rows are redundant constraints.

Many method identifying redundant constraints. A Heuristic method approach uses the intercept matrix [11, 12]. Gal proposed to classify constraints as redundant or non-redundant. Stojkovic-Stanimirovic proposed a method based on applying maximum and minimum principle [13, 14]. Llewellyn method identifies some special redundant constraints [15]. Llewellyn method comparing two constraints, then different constraints may be different conclusions [10]. In the paper, we give some examples to understand characteristics Heuristic method, Llewellyn method, and Stojkovic-Stanimirovic method.

2. Methods

Systems of positive linear inequalities constraints are as follow,

$$\boldsymbol{Ax} \leq \boldsymbol{b}$$
$$\boldsymbol{x} \geq \boldsymbol{0}. \quad (1)$$

where $\boldsymbol{A} \in R^{mxn}$ matrixs positive with $m \geq n$, $\boldsymbol{x} \in R^m$, $\boldsymbol{0} \in R^m$ and $\boldsymbol{b} \in R^m$.

Let S be feasibel region as follow,

$$S = \left\{ \boldsymbol{x} \in R^n \middle| \begin{matrix} \boldsymbol{Ax} \leq \boldsymbol{b} \\ \boldsymbol{x} \geq \boldsymbol{0} \end{matrix} \right\} \quad (2)$$

Meanwhile S_k be the feasible region without kth constraints is written as,

$$S_k = \left\{ \boldsymbol{x} \in R^n \middle| \begin{matrix} \sum_{j=1}^{n} \boldsymbol{A_i x} \leq b_i, i = 1, \dots, m, i \neq k \\ \boldsymbol{x} \geq 0, j = 1, \dots, n \end{matrix} \right\} \quad (3)$$

Constraint $\sum_{j=1}^{n} \boldsymbol{a_{kj} x_j} \leq b_k$ is redundant in the system inequality (1) if and only if $S = S_k$ [21]. Label **R** is assigned to redundant constraints. Strongly redundant and weakly redundant are clasification of redundant constraints by Jan Telgen [16]. Label **SR** is assigned to strongly redundant constraint. Its is defined as follows,

Definition 2.1. [16, 17] Constraint $\boldsymbol{A_k x} \leq b_k$ are strongly redundant if a constraint is redundant constraints and $\boldsymbol{A_k x} < b_k$ for some $\boldsymbol{x} \in S$.

Let weakly redundant will be referred to by label **WR**. Weakly redundant constraint is defined as follows,

Definition 2.2. [16, 17] Constraint $\boldsymbol{A_k x} \leq b_k$ is weakly redundant if a constraint is redundant constraints and $\boldsymbol{A_k x} = b_k$ for some $\boldsymbol{x} \in S$.

Consider of maximum linear programming,

Objective function : $\max z = \boldsymbol{cx}$

Subject to : $\boldsymbol{Ax} \leq \boldsymbol{b}$

$$\boldsymbol{x} \geq \boldsymbol{0}. \quad (4)$$

Where matrix $\boldsymbol{A} \in R^{mxn}$, vector $\boldsymbol{b} \in R^m$, vector $\boldsymbol{x} \in R^m$, vector $\boldsymbol{0} \in R^m$, and vector $\boldsymbol{c} \in R^n$.

We introduce three algorithms are algorithm 1 as Heuristic algorithm, algorithm 2 as Llewelyn algorithm and algorithm 3 as Stojkovic-Stanimirovic algorithm. There are as follows,

Algorithm 1 [10 - 12]

Input : $\boldsymbol{c} \in R^n$, $\boldsymbol{A} \in R_+^{mxn}$, $\boldsymbol{b} \in R^m$, $\boldsymbol{x} \in R_+^n$.

Output : Linier constraints without redundant constraints.

Step :

(1) Let I be $I = [1,2, \dots, m]$, while J be $J = [1,2, \dots, n]$.

(2) Let intercept matrix $\theta = [\theta_{ji}]$ with θ_{ji} as follows,

$$\theta_{ji} = \frac{b_i}{a_{ij}}: a_{ij} > 0 \; for \; j \in J, i \in I.$$

(3) Calculate $z_j - c_j$.

(4) Select $\beta_j = \min\{\theta_{ji}\} \; for \; j \in J, i \in I$.

(5) Calculate $z_j' - c_j' = \beta_j (z_j - c_j) \; for \; j \in J$.

(6) Select $z_k' - c_k' = \min \; \{z_j' - c_j'\}$.

(7) If $z_k' - c_k' \geq 0$, it does not has **R** and stop. Otherwise, remove index k element of J $(J = J - \{k\})$ and continue.

(8) Select $\theta_{kl} = \min\{\theta_{ki}\} = \beta_k$, remove index l element of I $(I = I - \{l\})$ and find p such that $\min\{\theta_{pl}\} = \beta_p$ for $p \in J$, so $J = J - \{p\}$.

(9) If $J = \{\emptyset\}$, then continue. Otherwise, go back to step 6.

(10) If $I = \{\emptyset\}$, then it does not has **R** and stop. Otherwise, $i \in I$ as **R** and stop.

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Algorithm 2 [10, 15]
Input : $\boldsymbol{A} \in R_+^{mxn}$, $\boldsymbol{b} \in R^m$, $\boldsymbol{x} \in R_+^n$.
Output : Linier constraints without redundant constraints.
Step :
Step 1: Compare two constraints of the k-th constraints and the s-th constraints. If b_k and $b_s \geq 0$, where $k = 1, \ldots, n$, $s = 1, \ldots, n$ and k ≠ s go to step 2. Otherwise, go to step 3.
Step 2: If $\frac{b_k}{a_{kj}} \geq \frac{b_s}{a_{sj}}$, $\forall j \in (1, \ldots, m)$ then the kth constraint is **R** and go to step 4.
Step 3: If $\frac{b_k}{a_{kj}} \leq \frac{b_s}{a_{sj}}$, $\forall j \in (1, \ldots, m)$ then the kth constraint is **R** and continue.
Step 4: If all constraints have been compared and stopped. Otherwise, go back to step 1.

Algorithm 3 [13]
Input : $\boldsymbol{c} \in R^n$, $\boldsymbol{A} \in R_+^{mxn}$, $\boldsymbol{b} \in R^m$, $\boldsymbol{x} \in R_+^n$.
Output : Linier constraints without redundant constraints.
Step :

1. Compute $d_{ij} = \left\{ \left[\frac{a_{ij}}{b_i c_j} \right] : i = 1,2, \ldots, m \text{ and } j = 1,2, \ldots, n \right\}$
2. if $\max_{1 \leq i \leq m} \min_{1 \leq j \leq n} d_{ij} = \min_{1 \leq i \leq n} \max_{1 \leq j \leq m} d_{ij}$, then it does not has **R**, and stop. Otherwise, continue.
3. Select k-th and l-th constraint so that if $d_{kj} \leq d_{ij}$, for all $j = 1,2, \ldots, k$-th constraint is **R**.

3. Result and Discussion

In section result and discussion, we give several examples to reveal some of the characteristics of Heuristic, Llewellyn and Stojkovic-Staminirovic method. Given Example 1 - Example 6 as follows,

Example 1

$$\max 6x_1 + 5x_2$$
$$x_1 + x_2 \leq 5$$
$$x_1 + x_2 \leq 6$$
$$x_1 + x_2 \leq 7$$
$$4x_1 + 2x_2 \leq 8$$
$$2x_1 + 4x_2 \leq 8$$
$$x_1, x_2 \geq 0.$$

Example 2

$$\max 3x_1 + 5x_2$$
$$5x_1 + 2x_2 \leq 10$$
$$5x_1 + 4x_2 \leq 20$$
$$5x_1 + 6x_2 \leq 30$$
$$4x_1 + 3x_2 \leq 12$$
$$4x_1 + 5x_2 \leq 20$$
$$x_1, x_2 \geq 0.$$

Example 3

$$\max 3x_1 + 5x_2$$
$$3x_1 + 5x_2 \leq 15$$
$$3x_1 + 7x_2 \leq 21$$
$$x_1 + x_2 \leq 8$$
$$7x_1 + 3x_2 \leq 21$$
$$7x_1 + 2x_2 \leq 14$$
$$x_1, x_2 \geq 0.$$

Example 4

$$\max 5x_1 + 7x_2$$

$$x_1 + x_2 \leq 5$$
$$x_1 + x_2 \leq 6$$
$$x_1 + x_2 \leq 7$$
$$4x_1 + 2x_2 \leq 8$$
$$2x_1 + 4x_2 \leq 8$$
$$x_1, x_2 \geq 0$$

Example 5

$$\max 5x_1 + 3x_2$$
$$3x_1 + 4x_2 \leq 12$$
$$6x_1 + x_2 \leq 6$$
$$x_1 + x_2 \leq 6$$
$$7x_1 + 8x_2 \leq 56$$
$$8x_1 + 7x_2 \leq 56$$
$$x_1, x_2 \geq 0.$$

Example 6

$$\max 5x_1 + 3x_2$$
$$x_1 + 5x_2 \leq 5$$
$$4x_1 + 3x_2 \leq 12$$
$$x_1 + x_2 \leq 4$$
$$x_1 + x_2 \leq 5$$
$$5x_1 + 7x_2 \leq 35$$
$$x_1, x_2 \geq 0.$$

Example 1 - Example 6 with the number of different **WR** and **SR** to find out methods of detecting **R** can identify **WR** and **SR** as **R**. Above examples are solved using several methods including Heuristic method (**A1**), Llewellyn method (**A2**), and Stojkovic-Staminirovic method (**A3**). That solving is presented in Table 1.

Table 1. Results identification of redundant constraints Example 1 - Example 6 uses redundant constraint detection methods.

No	Example	FR	WR	A1	A2	A3	R
1	Example 1	2 {4,5}	-	3 {1,2,3}	3 {1,2,3}	3{1,2,3}	3 {1,2,3}
2	Example 2	2 {1,4}	3{2,3,5}	-	3 {2,3,5}	-	3 {2,3,5}
3	Example 3	2 {1,5}	2{2,4}	1 {3}	3 {2,3,4}	-	3 {2,3,4}
4	Example 4	2 {4,5}	-	3 {1,2,3}	3 {1,2,3}	-	3 {1,2,3}
5	Example 5	2 {1,2}	1{3}	2 {4,5}	3 {3,4,5}	3{3,4,5}	3 {3,4,5}
6	Example 6	2 {1,2}	3{3,4,5}	-	3 {3,4,5}	-	3 {3,4,5}

FR is a constraint that forms a feasible region. Table 1 In example 1 has 5 inequality constraints (two **FR** are 4th & 5th constraint, and three **R** are 1st, 2nd & 3rd constraint). These three **R** are **SR**. Example 1 does not has **WR**. Its can be solved by Heuristic method, Llewellyn method, and Stojkovic-Staminirovic method. These methods can be detect three **R**. In Example 4 has same inequality constraints and different objective function with Example 1. Example 4 can be solved by Heuristic method and Llewellyn method. These methods can be detect three **R**. Meanwhile Example 4 cannot be solved by Stojkovic-Staminirovic method. The other example is analogous to Example 1 and Example 4.

4. Conclusion

Based on Example 1 – Example 6 the disadvantages of the Heuristic method is cannot detect **WR** as **R**. Meanwhile, Llewellyn method with many constraints will require more time and more work. Llewellyn method detects **R** with $n\ x\ (n-1)$ iterations. Stojkovic-Staminirovic method to detect whether a linear program problem has **R**. If it has **R** then there is $n\ x\ (n-1)$ iterations, it is the same as Llewellyn method because it compares the two constraints. Example 1 has same inequality constraints and different objective function with Example 4. Based on Example 1 and Example 4 show that the disadvantages of using the Stojkovic-Staminirovic method which is dependent on the objective function.

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