

# The Maximization of Wealth With Loss Aversion for an Insurance Company

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# The Maximization of Wealth With Loss Aversion for an Insurance Company

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The insurance company will invest the initial wealth on risk-free assets and risky assets to get the optimal portfolio. Investment decision maker is assumed to be loss aversion. The insurance company want to maximize the expected utility of wealth. In recent years, some researchers interested in optimal investment for a general insurance company. In this paper, the insurance company have a dynamic maximization model, so we translate the dynamic maximization model into an equivalent static optimization model with martingale method. Furthermore we solve the static optimization through Lagrange multiplier.

**Keywords:** insurance company, loss aversion, martingale method.

## 1. Introduction

Portfolio is investment in some financial instruments that can be traded on the Stock Exchange and Money Market.<sup>1</sup> The expected return refers to the value of a random variable one could expect if the process of finding the random variable could be repeated an infinite number of times. The uncertain condition cause the investor need to establish an optimal portfolio. The aim is to maximizing the expected return and minimize the risk. The concept of portfolio risk was first introduced by Harry M. Markowitz.

The theory of portfolio selection is generally based on the model of expected utility maximization (EUM). In recent years, some theories have been proposed to improve the weaknesses of the theory of EUM, one of them is Tversky et. al with Cumulative Prospect Theory or CPT.<sup>2</sup> The theory of CPT has also been used by Benartzi and Thaler and Bernard & Ghossoub in a static portfolio selection problem.<sup>3,4</sup> Some researchers like Berkelaar et. al studied the problem of dynamic portfolio selection in the CPT.<sup>5</sup> They compared the specifics two parts, namely the utility functions and optimal investment return on the loss averse investors.

Insurance Company is a financial services company with a product that is growing in Indonesia along with the growth of the national economy. Currently the insurance is more often perceived benefits both individual, group, community or business.

Based on previous research, studied the portfolio for the insurer.<sup>6</sup> In this paper, we need to change the model of maximizing the dynamic becomes static optimization models on the insurance company with the help of martingale method. The surplus process of insurance companies is modeled by Lévy process.

## 2. Problem Formulation

In this study, the problem formulation is how to change the model of maximizing the dynamic becomes static optimization models on the insurance company with the help of martingale method.

### 3. Methods

In this study, we use the martingale method to translate the model of maximizing the dynamic becomes static optimization models on the insurance company.

Define a martingale as follows.

If  $(\Omega, \mathcal{F}, P)$  is probability space. Stochastic process  $X: \Omega \times [0, \infty) \rightarrow \mathbb{R}$  is denote by  $X_n$  called martingale to filtration  $\{\mathcal{F}_n\}$  if

- (1)  $X_n$  measurable  $-\mathcal{F}_n$  for every  $n \geq 0$ .
- (2)  $E(X_n) < \infty$
- (3)  $E(X_{n+1} | \mathcal{F}_n) = X_n$  for every  $n \geq 0$

### 4. Results and Discussion

#### 4.1 Trading Strategies

The insurance company invests in the stock market where assets at  $n + 1$  are traded continuously when  $[0, T]$ . One is a riskless asset with price  $(S_0)$  is given by

$$dS_0(t) = S_0(t) \cdot r(t) dt \quad (4.1)$$

with  $S_0(0) = 1$  and  $r(t)$  is interest rate.

While  $n$  risk asset with price  $(S_i)$  is given by

$$dS_i(t) = S_i(t) \left[ b_i(t) dt + \sum_{j=1}^n \sigma_{ij}(t) dB_j(t) \right] \quad (4.2)$$

$$S_i(0) > 0, j = 1, 2, \dots, n$$

Where  $B(t)$  is Brownian motion on a probability space  $(\Omega, \mathcal{F}, \mathbb{P})$ ,  $b(t)$  is drift coefficient, and  $\sigma(t)$  is volatility.

#### 4.2 Cramér –Lundberg Process

Cramér - Lundberg Process can be used to determine the surplus of insurance company that given by [7]

$$U(t) = x + \int_0^t \alpha(s) ds - S(t) = x + \int_0^t \alpha(s) ds - \sum_{i=1}^{K(t)} Z_i \quad (4.3)$$

where  $x > 0$  is initial wealth of insurance company,  $\alpha(s) > 0$  is the premium rate at time  $t$ ,  $S(t)$  in a compound Poisson process, and  $K(t)$  is number of claims occurring in time interval  $[0, t]$ .

Stochastic process of (4.3) is  $S(t)$  with probability space  $\{\Omega, \mathcal{F}, \mathbb{P}\}$  that can describe by.<sup>7</sup>

- (i)  $\mathbb{P}$  is a filtration  $\sigma$  and  $0 \leq \sigma_1 \leq \sigma_2 \leq \dots$  is timing of claims payments. If  $\sigma_0 = 0$ , random variable  $T_n = \sigma_n - \sigma_{n-1}$  ( $n = 1, 2, \dots$ ) non-negative.
- (ii)  $K(t) = \sup\{n: \sigma_n \leq t\}$ ,  $t \geq 0$  is number of claims occurring in time interval  $[0, t]$ . The relationship between time  $\{\sigma_0, \sigma_1, \dots\}$  and counting process  $\{K(t), t \geq 0\}$  is given by  $\{K(t) = n\} = \{\sigma_n \leq t < \sigma_{n+1}\}$ ,  $n = 0, 1, \dots$
- (iii) Sequence of non-negative random variable  $\{Z_n, n = 1, 2, \dots\}$  is the amount of claims paid by insurance companies. Sequence of  $\{Z_n\}$  is independent, so counting of number of claims up to  $t$  is given by

$$S(t) = \sum_{i=1}^{K(t)} Z_i, t \geq 0$$

### 4.3 Lévy Process

Lévy process has an important role in the financial world. In actuarial, this process is used in the calculation of risk insurance and re-insurance.

#### Definition 4.1

Let  $(\Omega, \mathcal{F}, \mathbb{P})$  be a filtered probability space, where  $\mathcal{F}$  is filtration, and stochastic process of  $L$  is called a Lévy process if the following conditions are satisfied.<sup>8</sup>

- $L$  has independent increments, i.e.  $L_t - L_s$  is independent of  $\mathcal{F}_s$  for any  $0 \leq s < t < T$ .
- $L$  has stationary increments, for any  $0 \leq s, t < T$ , the distribution of  $L_{t+s} - L$  does not depend on  $t$ .
- $L$  is stochastically continuous for every  $0 \leq t \leq T$ , and  $\varepsilon > 0 \lim_{s \rightarrow t} P(|L_t - L_s| > \varepsilon) = 0$

Denote  $L(t)$  is compensated compound Poisson process, i.e.

$$L(t) := S(t) - E(Z) \int_0^t \lambda(s) ds$$

Then  $L(t)$  is a 1-dimensional compensated pure Lévy process defined in the probability space  $(\Omega, \mathcal{F}, \mathbb{P})$  and  $L(t)$  is a martingale.<sup>9</sup>

Let  $N$  denote Poisson random measure (or jump measure) of  $L$  and  $\nu$  denote the Lévy measure that satisfies

$$\nu(0) = 0, \quad \int_{\mathbb{R}} (1 + |x|^2) \nu(dx) < \infty$$

Intuitively speaking, the Lévy measure describes the expected number of jumps of certain height in a time interval of length 1. For the risk process model (3.3), we have  $\nu(dx) = \lambda F(dx)$ .

$L$  has Lévy decomposition as follows.<sup>10</sup>

$$L(t) = \int_0^t \int_{\mathbb{R}} z (N(ds, dz) - \nu(dz) ds)$$

By using the Lévy process, so that the surplus (3.3) can be given by

$$U(t) = x + \int_0^t c(s) ds - L(t)$$

where  $c(s) = \alpha(t) - \lambda(t)E(Z)$ .

### 4.4 Optimal Portfolio Model

In this section we will change the model of maximizing the dynamic becomes static optimization models on the insurance company. The insurer would invest in the  $n + 1$  assets continuously. We denote  $X(t)$  is his wealth at time  $t$ . Then  $X(t)$  satisfies the following stochastic differential equation:

$$\begin{cases} dX(t) = (X(t)r(t) + \pi(t)^T (b(t) - r(t)) + c(t))dt + \pi(t)^T \sigma(t)dB(t) - dL(t) \\ X(0) = x \end{cases} \quad (4.4)$$

Following utility maximization criterion, the model of maximizing the dynamic of optimal portfolio for an insurer can be formulated as follows:

$$\begin{cases} \max_{\pi \in \Pi} U(X(T)) \\ X(t) \text{ satisfies (4.4)} \\ X(t) \geq 0, \forall t \in [0, T] \end{cases} \quad (4.5)$$

According to equation (4.4) we can see that  $X(t)$  is not a martingale under  $\mathbb{P}$ , then we need to define the discount factor so that the process  $X(t)$  be a martingale. Discount factor is defined as follows.

$$H(t) = \exp \left\{ - \int_0^t r(s) ds - \frac{1}{2} \int_0^t \|\theta(s)\|^2 ds - \int_0^t \theta(s) dB(s) + \int_0^t \int_{\mathbb{R}} \ln p(s, z) N(ds, dz) + \int_0^t \int_{\mathbb{R}} (1 - p(s, z)) v(dz) ds \right\} \quad (4.6)$$

where  $\int_{\mathbb{R}} (1 - p(s, z)) v(dz) = c(t)$  and  $\theta(t) = \sigma^{-1}[b_i(t) - r(t)]$  is market price of risk.

We have the following conclusion.

**Proposition 4.1** If  $H(t)$  is defined by (4.6) for  $t \in [0, T]$ , then  $H(t)X(t)$  is a martingale under the probability measure  $\mathbb{P}$ .

Proof:

According to (3.4), so we can write  $X(t)$  in the differential form.

$$\begin{aligned} dX(t) &= (X(t)r(t) + \pi(t)^T (b(t) - r(t)) + c(t)) dt + \pi(t)^T \sigma(t) dB(t) - dL(t) \\ &= X(t)r(t) dt + \pi(t)^T (b(t) - r(t)) dt + c(t) dt + \pi(t)^T \sigma(t) dB(t) - dL(t) \\ &= X(t)r(t) dt + \pi(t)^T \sigma(t) \left[ \left( \frac{b(t) - r(t)}{\sigma(t)} \right) dt + dB(t) \right] + c(t) dt - dL(t) \\ &= X(t)r(t) dt + \pi(t)^T \sigma(t) [\theta(t) dt + dB(t)] + c(t) dt - dL(t) \end{aligned}$$

We assumed  $dB(t) = \theta(t) dt + dB(t)$ , so

$$dX(t) = X(t)r(t) dt + \pi(t)^T \sigma(t) dB(t) + c(t) dt - dL(t) \quad (4.7)$$

Then, from definition of (4.6) we can write  $H(t)$  in the differential form

$$\begin{aligned} dH(t) &= -r(t) H(t) dt + \frac{1}{2} \|\theta(t)\|^2 H(t) dt - \frac{1}{2} \|\theta(t)\|^2 H(t) dt \\ &\quad - \theta(t) H(t) dB(t) + p(t, z) N(dt, dz) - N(dt, dz) - p(t, z) v(dz) dt \\ &\quad + v(dz) dt \\ &= -r(t) H(t) dt - \theta(t) H(t) dB(t) + (p(t, z) - 1) (N(dt, dz) - v(dz) dt) H(t) \\ &= H(t) [-r(t) dt - \theta(t) dB(t) + (p(t, z) - 1) \tilde{N}(dt, dz)] \end{aligned} \quad (4.8)$$

To show that process discounted of  $X(t)$  is a martingale under  $\mathbb{P}$ , so we can write the differential form as follows

$$\begin{aligned} d(H(t)X(t)) &= dH(t)X(t) + H(t)dX(t) + d[H(t), X(t)] \\ &= H(t)X(t)[-r(t) dt - \theta(t) dB(t)] \\ &\quad + H(t)[X(t)r(t) dt + \pi(t)^T \sigma(t) dB(t)] \\ &\quad + H(t)X(t)(p(t, z) - 1)N(dt, dz) \\ &\quad - H(t)X(t)(p(t, z) - 1)v(dz)dt - H(t)p(t, z)N(dt, dz) \\ &\quad + H(t)p(t, z)v(dz)dt \end{aligned}$$

$$\begin{aligned}
&= -r(t) H(t) X(t) dt - H(t) X(t) \theta(t) dB(t) \\
&\quad + r(t) H(t) X(t) dt + \pi(t) \sigma(t) H(t) dB(t) \\
&\quad + H(t) N(dt, dz) [X(t)(p(t, z) - 1) - p(t, z)] \\
&\quad - H(t) v(dz) dt [X(t)(p(t, z) - 1) - p(t, z)] \\
&= -H(t) X(t) \theta(t) dB(t) + \pi(t) \sigma(t) H(t) dB(t) \\
&\quad + H(t) [X(t)(p(t, z) - 1) - p(t, z)] (N(dt, dz) - v(dz) dt) \\
&= H(t) [-X(t) \theta(t) + \pi(t) \sigma(t)] dB(t) + H(t) [X(t)(p(t, z) - 1) - p(t, z)] \tilde{N}(dt, dz)
\end{aligned} \tag{4.9}$$

We pray for integration on both sides of an equation (4.9) and get

$$\begin{aligned}
H(t)X(t) &= X(0) + \int_0^t H(s) [-X(s)\theta(s) + \pi(s)\sigma(s)] dB(s) \\
&\quad + \int_0^t H(s) [X(s)(p(s, z) - 1) - p(s, z)] \tilde{N}(ds, dz)
\end{aligned} \tag{4.10}$$

Based on the above equation, since every term is an integral Ito, then  $H(t)X(t)$  is a martingale under the probability measure  $\mathbb{P}$ .

■

Then, we need to change the model of maximizing the dynamic (4.5) becomes static optimization model that satisfied Theorem 4.1.

**Theorem 4.1** Let  $\eta \geq 0$  be an  $\mathcal{F}_T$ -measurable random variable, then for a given initial wealth  $x$  satisfying  $\mathbb{E}[H(T)\eta] = x$  there exist a portfolio process such that  $\pi(t) \in \Pi$ ,  $t \in [0, T]$  and  $X^\pi(T) = \eta$ .

Proof:

Define a negative martingale  $M(t) = \mathbb{E}[H(T)\eta | \mathcal{F}_t]$ .

$M(t)$  is  $\mathbb{P}$ -martingale so according to the martingale representation theorem, there exist a predictable process  $\psi(t): \Omega \times [0, T] \mapsto \mathbb{R}$  and  $\varphi(t): \Omega \times [0, T] \mapsto \mathbb{R}^n$ ,  $0 \leq t \leq T$ .<sup>11</sup> The theorem is satisfying

$$\begin{aligned}
&\int_0^T \|\varphi(s)\|^2 ds < \infty, a.s \\
&\int_0^T \int_{\mathbb{R}} |\psi(t, z)|^2 v(dz) dt < \infty, a.s
\end{aligned}$$

Such that

$$\begin{aligned}
M(t) &= M_0 + \int_0^t \varphi(s) dB(s) + \int_0^t \int_{\mathbb{R}} \psi(t, z) \tilde{N}(ds, dz) \\
&= x + \int_0^t \varphi(s) dB(s) + \int_0^t \int_{\mathbb{R}} \psi(t, z) \tilde{N}(ds, dz)
\end{aligned} \tag{4.11}$$

Comparing  $dB(t)$ -term, and  $\tilde{N}(dt, dz)$ -term respectively in (4.11) and (4.10) on the right side, it is reasonable conjecture

$$X(0) = M(0) = \mathbb{E}[H(T)\eta] \tag{4.12}$$

$$\pi(t) = (\sigma(t)^T)^{-1} [\varphi(t)H(t)^{-1} + \theta(t)X(t)] \tag{4.13}$$

$$\psi(t, z) = H(t) [X(t)(p(t, z) - 1) - p(t, z)] \tag{4.14}$$

while on the left side, we have

$$H(t)X(t) = M(t) = \mathbb{E}[H(T)\eta | \mathcal{F}_t]$$

$$H(t)X(t) = H(T)\eta$$

$$X(t) = \eta \quad (4.15)$$

Then we need to check if the process  $\pi(t)$  defined in (4.13) is an admissible portfolio. Let  $(\sigma(t)^T)^{-1} = (c_{ij}(t))_{n \times n}$ ,  $\varphi(t) = (\varphi_1(t), \dots, \varphi_n(t))^T$ , and  $\theta(t) = (\theta_1(t), \dots, \theta_n(t))^T$ .

We define some notations:

$$\|f(t)\|_\infty = \max_{0 \leq t \leq T} |f(t)| \text{ dan } \|f(t)\|_2 = \left( \int_0^T |f(t)|^2 dt \right)^{\frac{1}{2}}$$

We have,

$$\begin{aligned} \int_0^T |\pi_i(t)| dt &= \int_0^T |(\sigma(t)^T)^{-1} [\varphi(t)H(t)^{-1} + \theta(t)X(t)]| dt \\ &= \int_0^T |(c_{ij}(t))_{n \times n} [\varphi_j(t)H(t)^{-1} + \theta_j(t)X(t)]| dt \\ &\leq \int_0^T |(c_{ij}(t))_{n \times n} \varphi_j(t)H(t)^{-1}| dt + \int_0^T |(c_{ij}(t))_{n \times n} \theta_j(t)X(t)| dt \\ &\leq \int_0^T \left| H(t)^{-1} \sum_{j=1}^n c_{ij}(t) \varphi_j(t) \right| dt + \int_0^T \left| X(t) \sum_{j=1}^n c_{ij}(t) \theta_j(t) \right| dt \\ &\leq \|H^{-1}\|_\infty \|c_{ij}(t)\|_\infty \|\varphi_j(t)\|_2 + \|X(t)\|_\infty \|c_{ij}(t)\|_\infty \|\theta_j(t)\|_2 \end{aligned}$$

$< \infty$  a.s

■

The last inequality follows from the uniformly bounded conditions.

According to Theorem 4.1, any  $\mathcal{F}_t$ -measurable random variable  $\eta \geq 0$  with  $\mathbb{E}[H(T)\eta] = x$  can be financed via trading an admissible portfolio  $\pi$  such that  $X^\pi(T) = \eta$ . So to determine the optimal portfolio process  $\pi^*(t)$  in the dynamic maximization models (4.5), which depends on the time variable  $t$ , it is sufficient to maximize over all possible random variable  $\eta$ 's. That is to say, the dynamic maximization models (4.5) is equivalent to the following static optimization models:

$$\begin{cases} \max_{\eta \geq 0} \mathbb{E}[U(\eta)] \\ \text{s. t. } \mathbb{E}[H(T)\eta] \leq x \end{cases} \quad (4.15)$$

#### 4. Conclusion

In conclusion, this study shows that with martingale method we have optimal portfolio process  $\pi^*(t)$  in the dynamic maximization models, which depends on the time variable  $t$ , it is sufficient to maximize over all possible random variable  $\eta$ 's. It means, the dynamic maximization problem is equivalent to the following static optimization models. Such that, the static optimization models on the insurance company is

$$\begin{cases} \max_{\eta \geq 0} \mathbb{E}[U(\eta)] \\ \text{s. t. } \mathbb{E}[H(T)\eta] \leq x \end{cases}$$

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