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Bi-Objective Model Predictive Controller for Supply Chains Management Without Delay

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In this paper, we design bi-objective model predictive controller for inventory management in supply chains without any delay in production or shipments. The objectives used are obtained from model predictive control and economic model predictive control. Adaptive Weighted Sum (AWS) method is used to design a bi-objective optimization problem by combining these two control strategies and weighting each of the respective strategy based on a subjective perspective. The acquired control is then compared to model predictive control and economic model predictive control in a numerical simulation. Based on the results from the simulation, it can be seen that the control obtained through AWS method could stabilize a system with more cost-effective inputs when it is compared with model predictive control. The results also show that the control can stabilize a wider range of initial state when it is compared to economic model predictive control.

Keywords: Model Predictive Control, Economic Model Predictive Control, Adaptive Weighted Sum, Biobjective Optimization.

1. INTRODUCTION

Control theory has been a peculiar area of research, especially in its application within supply chains management.[1,2] An important consideration in process control is the stability of the closed-loop system under the proposed control. Despite that, the stability of rolling horizon optimization management has not been researched sufficiently.

Model predictive control (MPC) is a rolling horizon optimization control method with guaranteed stability properties.[3] Over the past decade, many different frameworks for control management have been studied.[4–6] In spite of being a fairly mature field, MPC has trouble in accounting for the economic costs the control would impose. The recently developed economic model predictive control (EMPC),[7,8] on the other hand, fails to account for the stability of the system for the sake of optimizing the economic costs required for the system to operate.

Multiple, possibly conflicting, objective functions often present when one is designing a control. For example, one may want to maximize the performance of a system while minimizing its cost. The traditional approach in formulating multiobjective optimization problems is by using a weighted-sum approach. Despite its existence, the approach has drawn many criticisms in regards to its drawbacks.[9] A new method recently has been proposed in 2005, a method that is called Adaptive Weighted Sum (AWS) method.

In this paper, we analyze the utilization of AWS method developed by Kim and De Weck[10] and its application in designing a biobjective control method as presented by Subramanian, Rawlings, and Maravelias.[11] The control method presented is able to simultaneously account for the stability of a system and the economic costs imposed by the operation of the process in a supply chain dynamic with delays present. Here, we design a biobjective control for a supply chain with no shipment/production delays. The obtained control is applied on a two-node, single product supply chain in a numerical simulation. The results of proposed control is compared with MPC and EMPC.

2. MATHEMATICAL MODELING OF SUPPLY CHAINS MANAGEMENT

We consider the following linear model

$$\boldsymbol{x}(k+1) = A\boldsymbol{x}(k) + B\boldsymbol{u}(k) + B_d\boldsymbol{d}_s \tag{1}$$

in which $\boldsymbol{x} \in \mathbb{R}^n$ is the system state, $\boldsymbol{u} \in \mathbb{R}^m$ is the manipulated input, and $\boldsymbol{d}_s \in \mathbb{R}^d$ is the disturbance to the system. We assume that the system (A, B) is stabilizable.

The states and inputs are constrained as follows:

$$\boldsymbol{x} \in \mathbb{X}, \quad \boldsymbol{u} \in \mathbb{U} \tag{2}$$

The optimal steady state problem for the nominal demand d_s is defined as:[11]

$$\begin{aligned} \min_{\boldsymbol{x},\boldsymbol{u}} \quad & l(\boldsymbol{x}, \boldsymbol{u}) \\ \text{s.t.} \quad & \boldsymbol{x} = A\boldsymbol{x} + B\boldsymbol{u} + B_d\boldsymbol{d}_s, \\ & x \in \mathbb{X}, \quad u \in \mathbb{U} \end{aligned} \tag{3}$$

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in which $l(\boldsymbol{x}, \boldsymbol{u})$ is the stage cost of implementing input $\boldsymbol{u}$ from state $\boldsymbol{x}$. The optimal steady state acquired is denoted by $(\boldsymbol{x}_s, \boldsymbol{u}_s; \boldsymbol{d}_s)$.

The states in this model consist of inventory and backorder. The inputs consist of shipments, productions, and orders. Because there are no delays in this system, there is no need to remodel the mathematical system with additional states and redefine the parameters.

In order to ensure the closed-loop stability of the controller, we follow the method presented by Subramanian, Rawlings, and Maravelias[11] as we make the following assumptions:

Assumption 1. *The constraint set* $\mathbb{X}$ *is convex and closed. The constraint set* $\mathbb{U}$ *is convex and compact. The optimal steady state* $(\boldsymbol{x}_s, \boldsymbol{u}_s; \boldsymbol{d}_s)$ *is such that* $\boldsymbol{x}_s \in \mathbb{X}$ *and* $\boldsymbol{u}_s \in \mathbb{U}$.

Assumption 2. *There exists* $(\boldsymbol{x}_s, \boldsymbol{u}_s; \boldsymbol{d}_s)$ *and* $\boldsymbol{\lambda}_s$ *so that*
(a) $(\boldsymbol{x}_s, \boldsymbol{u}_s; \boldsymbol{d}_s)$ *is a unique solution of* (3),
(b) *The multiplier* $\boldsymbol{\lambda}_s$ *is such that* $(\boldsymbol{x}_s, \boldsymbol{u}_s; \boldsymbol{d}_s)$ *uniquely solves* (4)

$$\begin{aligned} \min_{x,u} \quad & l_E(x,u) - \lambda_s^T[x - (Ax + Bu + B_d d)] \\ \text{s.t} \quad & \boldsymbol{x} \in \mathbb{X}, \quad \boldsymbol{u} \in \mathbb{U} \end{aligned} \tag{4}$$

(c) *The system* $\boldsymbol{x}(k+1) = A\boldsymbol{x}(k) + B\boldsymbol{u}(k) + B_d\boldsymbol{d}_s$ *is strictly dissipative with respect to the supply rate* $s(\boldsymbol{x}, \boldsymbol{u}) = l_E(\boldsymbol{x}, \boldsymbol{u}) - l_E(\boldsymbol{x}_s, \boldsymbol{u}_s)$ *and storage function* $\lambda(\boldsymbol{x}) = \boldsymbol{\lambda}_s^T \boldsymbol{x}$. *That is, there exists a positive definite function* $\rho(\cdot)$ *such that for all* $(\boldsymbol{x}, \boldsymbol{u}) \in \mathbb{X} \times \mathbb{U}$:

$$\boldsymbol{\lambda}_s^T (A\boldsymbol{x} + B\boldsymbol{u} + B_d\boldsymbol{d}_s - \boldsymbol{x}) \le -\rho(\boldsymbol{x} - \boldsymbol{x}_s) + s(\boldsymbol{x}, \boldsymbol{u}) \tag{5}$$

3. BIOBJECTIVE MODEL PREDICTIVE CONTROLLER DESIGN

In this paper, we use a biobjective stage cost, which is a weighted sum of the economic stage cost used in EMPC and the tracking stage cost used in MPC. The bi-objective stage cost is:

$$l(\boldsymbol{x}, \boldsymbol{u}) = \frac{\omega}{s_E} l_E(\boldsymbol{x}, \boldsymbol{u}) + \frac{(1-\omega)}{s_T} l_T(\boldsymbol{x}, \boldsymbol{u}; z_t) \tag{6}$$

in which the parameter $\omega \in [0, 1]$ is a relative weighting provided to the economic and tracking stage costs. The function $l_E(\boldsymbol{x}, \boldsymbol{u})$ is the economic cost of the system. Assuming that the economic cost is linear, the stage cost is defined as:

$$l_E(\boldsymbol{x}(j), \boldsymbol{u}(j)) = q^T \boldsymbol{x}(j) + r^T \boldsymbol{u}(j) \tag{7}$$

where q' and r' are vectors which represents the effect of the current state and input to the economic cost of the system. The function and $l_T(\boldsymbol{x}, \boldsymbol{u}; z_t)$ is a tracking stage cost which penalizes deviations from a chosen steady-state $z_t = (\boldsymbol{x}_t, \boldsymbol{u}_t)$. The tracking stage cost is defined as:

$$\begin{aligned} & l_T(\boldsymbol{x}(k), u(k); z_t) \\ & \quad = (\boldsymbol{x}(k) - \boldsymbol{x}_t)^T Q(\boldsymbol{x}(k) - \boldsymbol{x}_t) + (\boldsymbol{u}(k) - \boldsymbol{u}_t)^T R(\boldsymbol{u}(k) - \boldsymbol{u}_t) \end{aligned} \tag{8}$$

in which matrices Q and R are positive semi-definite matrices which guides and maintain states and inputs to their respective steady-state.

The parameters s_T, s_E are scaling parameters obtained using the utopia and nadir points of the individual stage costs.[10] Denote $\boldsymbol{z} = (x, u)$. We solve

$$z_E = \underset{z \in X \times U}{\operatorname{argmin}}\, l_E(\boldsymbol{x}, \boldsymbol{u}), \quad z_T = \underset{z \in X \times U}{\operatorname{argmin}}\, l_T(\boldsymbol{x}, \boldsymbol{u}; z_t) \tag{9}$$

The utopia point is the best possible costs that can be attained for both the cost functions:

$$J^U = (l_E(z_E), l_T(z_T; z_t)) \in \mathbb{R}^2 \tag{10}$$

The nadir point is the cost attained by one stage cost at the optimal solution of the other stage cost:

$$J^N = (l_E(z_T), l_T(z_E; z_t)) \in \mathbb{R}^2 \tag{11}$$

The parameters s_T, s_E are then defined as:

$$(s_E, s_T) = J^N - J^U \tag{12}$$

In this section we will use the terminal constraint formulation as the centralized MPC problem.[11] The problem is defined as follows:

$$\begin{aligned} \min_{\boldsymbol{u}(0), \boldsymbol{u}(1), \ldots, \boldsymbol{u}(N-1)} \quad & V_N(\boldsymbol{u}(0), \boldsymbol{u}(1), \ldots, \boldsymbol{u}(N-1); \boldsymbol{x}_0) \\ \text{s.t} \quad & \boldsymbol{x}(0) = \boldsymbol{x}_0 \\ & \boldsymbol{x}(j+1) = A\boldsymbol{x}(j) + B\boldsymbol{u}(j) + B_d\boldsymbol{d} \\ & \boldsymbol{x}(j) \in \mathbb{X} \\ & \boldsymbol{u}(j) \in \mathbb{U} \\ & \boldsymbol{x}(N) = \boldsymbol{x}_s \\ & j \in \{0, 1, \ldots, N-1\} \end{aligned} \tag{13}$$

in which the cost function $V_N(\boldsymbol{u}(0), \boldsymbol{u}(1), \ldots, \boldsymbol{u}(N-1); \boldsymbol{x}_0)$ is the sum of stage costs

$$V_N(\boldsymbol{u}(0), \boldsymbol{u}(1), \ldots, \boldsymbol{u}(N-1); \boldsymbol{x}_0) = \sum_{j=0}^{N-1} l(\boldsymbol{x}(j), \boldsymbol{u}(j)) \tag{14}$$

The control horizon is denoted by N.

The control is obtained by computing $\boldsymbol{u}(0), \boldsymbol{u}(1), \ldots, \boldsymbol{u}(N-1)$ which optimally minimizes (13). For computational purposes, one could divide the optimization problem into N problems by individually searching for $\boldsymbol{u}(j)$ which minimizes $l(\boldsymbol{x}(j+1), \boldsymbol{u}(j))$.

The control law $\kappa(\boldsymbol{x})$ is the first input in the optimal solution to optimization problem (13). The admissible region χ_N is given by

$$\begin{aligned} \chi_N := \{ & \boldsymbol{x} \in X \mid \exists\, (\boldsymbol{u}(0), \boldsymbol{u}(1), \ldots, \boldsymbol{u}(N-1)) \in \mathbb{U}^N, \\ & \text{s.t (13) is feasible} \} \end{aligned} \tag{15}$$

According to Subramanian, Rawlings, and Maravelias,[11] as long as Assumption 1 and 2 hold, we can ensure the exponential stability for the controller that solves problem (13). We use the theorem for Lyapunov function with terminal constraint, which states that the steady-state solution of the closed loop system $\boldsymbol{x}(k+1) = A\boldsymbol{x}(k) + B\kappa(\boldsymbol{x}) + B_d\boldsymbol{d}_s$ is asymptotically stable with χ_N as the region of attraction. The Lyapunov function is[8]

$$\tilde{V}(\boldsymbol{x}) = V_N^0(\boldsymbol{x}) + \boldsymbol{\lambda}_s^T[\boldsymbol{x} - \boldsymbol{x}_s] - N l_E(\boldsymbol{x}_s, \boldsymbol{u}_s) \tag{16}$$

4. RESULTS AND DISCUSSION

In this section, we will observe the biobjective control as stated in (13) under a numerical simulation. The following results of the simulation then will be compared to MPC and EMPC.

As shown in Figure 1, we use a two-stage, single-product supply chain with a retailer and a manufacturer. The manufacturing

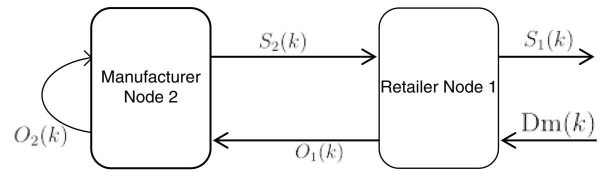

Fig. 1. Two-stage supply chain.

delay and the shipment delay are assumed to be nonexistent. The retailer responds to customer demand Dm by shipping S_1 units to the customer and ordering O_1 to the manufacturer. These decisions are based on the retailer's inventory and backorder. Similarly, the manufacturer responds to retailer's orders by making shipments S_2 and production O_2. The dynamics for the supply chain is:

$$\begin{bmatrix} Iv_1(k+1) \\ BO_1(k+1) \\ Iv_2(k+1) \\ BO_2(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Iv_1(k) \\ BO_1(k) \\ Iv_2(k) \\ BO_2(k) \end{bmatrix} + \begin{bmatrix} -1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} S_1(k) \\ O_1(k) \\ S_2(k) \\ O_2(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} Dm \tag{17}$$

The input constraints consist of non-negativity and the maximum production/shipping between nodes. Similarly, the state constraints consist of non-negativity constraints and the maximum inventory/backorder capacity. The constraints of this system are represented as:

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \le \boldsymbol{u}(k) \le \begin{bmatrix} 60 \\ 60 \\ 60 \\ 60 \end{bmatrix} \tag{18}$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \le \boldsymbol{x}(k) \le \begin{bmatrix} 50 \\ 50 \\ 50 \\ 50 \end{bmatrix} \tag{19}$$

The parameters used are $Q = 10^{-1}\text{diag}(1, 1, 1, 1)$, $R = \text{diag}(1, 1, 1, 1)$, $x_t = (35, 0, 40, 0)$, $u_t = (10, 10, 10, 10)$. The economic costs are $(30, 5, 20, 5)$, $r = (20, 1, 10, 150)$. The nominal demand is $d_s = 10$.

In this section we use the control in (13) as the centralized MPC optimization problem. We obtain the following stage cost:

$$l(\boldsymbol{x}, \boldsymbol{u}) = \frac{\omega}{1.85\times 10^3}(q'\boldsymbol{x} + r'\boldsymbol{u}) + \frac{1-\omega}{282.5}((\boldsymbol{x}-\boldsymbol{x}_t)'Q(\boldsymbol{x}-\boldsymbol{x}_t) + (\boldsymbol{u}-\boldsymbol{u}_t)'R(\boldsymbol{u}-\boldsymbol{u}_t)) \tag{20}$$

in which $\omega \in [0, 1]$.

In Figure 2, we plot the closed-loop response for three different values of ω (0.2, 0.6, and 1). Alongside, we plot the response

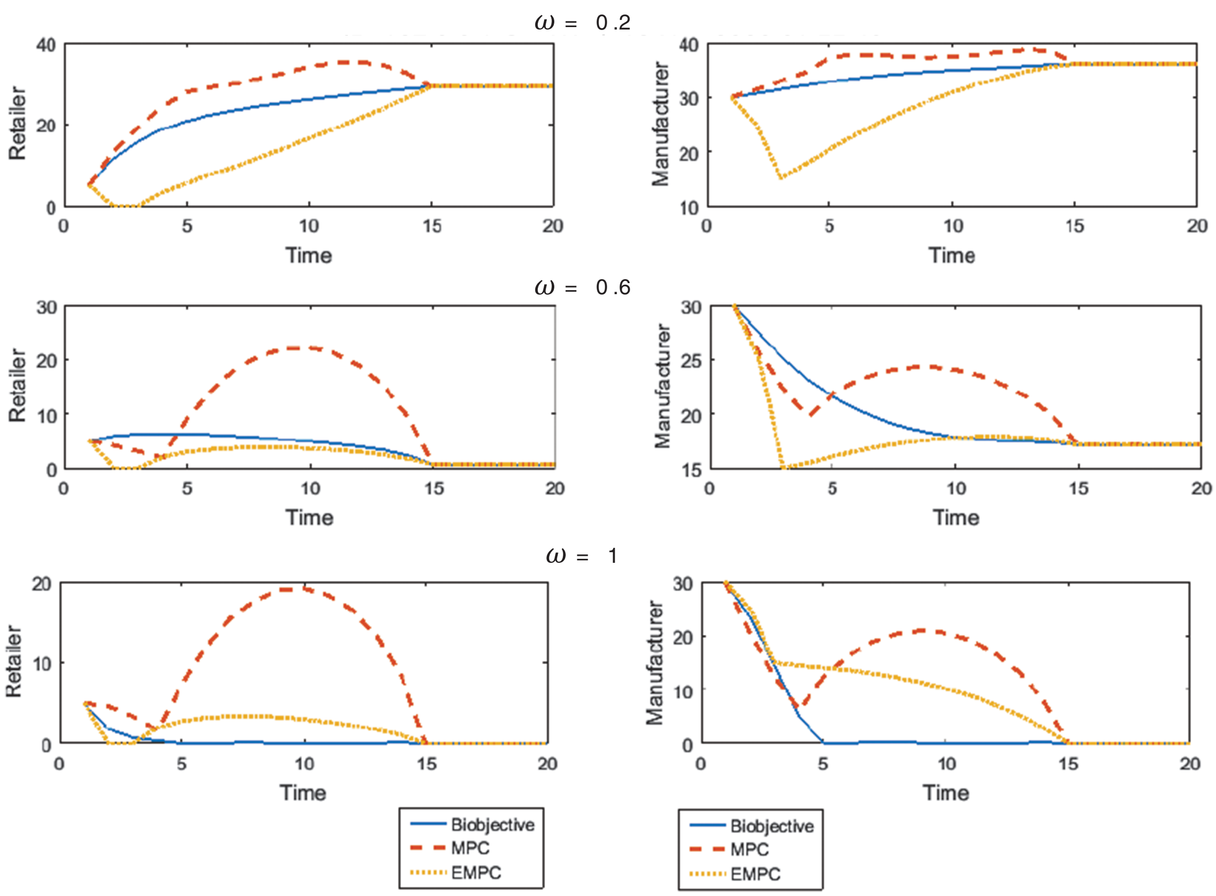

Fig. 2. Comparison of closed-loop response using an MPC stage cost and a biobjective stage cost for different values of ω.

Table I. Economic cost of implementing controller.

ω	Bi-objective ($\times 10^4$)	MPC ($\times 10^4$)	EMPC ($\times 10^4$)
0.2	6.9467	7.3862	6.3440
0.6	4.2897	4.9515	4.2324
1	3.2735	4.2305	3.2735

for MPC which tracks to the same steady state. That is, we use the stage cost as $l_T(\boldsymbol{x}, \boldsymbol{u}; z_s)$ in which z_s is the steady state of the biobjective formulation.

From Figure 2, we could observe how biobjective MPC could stabilize a system as effective as MPC. The amount of inventory stocked in MPC is bigger than the amount of inventory in biobjective MPC, noticeably so in biobjective MPC where ω is closer to 1. When ω is closer to 0, the biobjective control puts more focus on stabilizing the system rather than minimizing economic costs. When ω is closer to 1, the control focuses more on economically optimizing the system, less on stabilizing the system.

In Table I, we compare the economic cost incurred in using biobjective MPC ($l(\boldsymbol{x}, \boldsymbol{u})$), MPC to the steady state of the biobjective MPC ($l_T(\boldsymbol{x}, \boldsymbol{u}; z_s)$), and EMPC ($l_E(\boldsymbol{x}, \boldsymbol{u})$) with the terminal constraint of the steady state of the biobjective MPC. Here, we observe how under three different values of ω, biobjective MPC managed to stabilize the system on a lower economic cost when compared to MPC. We could also observe how EMPC, despite being able to stabilize the system on the lowest economic cost, fails to represent the perspective of the system manager to be prepared on unwanted risks and damages. Biobjective MPC is able to stabilize the system with lesser risks than EMPC.

5. CONCLUDING REMARKS

The purpose of this paper is to demonstrate a possible control design which can capture a system manager's subjective perspective under a single parameter. In the bi-objective MPC, the control is less prepared to face unpredicted disturbance if it is too focused on minimizing costs. Consequently, the bi-objective control is more prepared to face unprecedented risks if it focuses more on stabilizing the system, imposing more operational costs to the system. The exponential stability ensured the asymptotical stability of the bi-objective MPC as long as the required assumptions hold.

From the numerical simulation, we can observe that when economic information is available to the controller, it follows an economically attractive transient while stabilizing the steady state. In other words, the bi-objective formulation is better than MPC as it could stabilize to the steady state via a cost-effective transient. We can also observe how the bi-objective controller is able to represent the risk-averse nature of a system manager better when compared to EMPC. This makes bi-objective MPC more desirable as using EMPC poses risks to the system.

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