Deterministic Dynamic Programming Approach to Solve an Integrated Dynamic Supplier Selection and Inventory Control

Sutrisno,[a]* Widowati,[b] R. Heru Tjahjana[c]

[a,b,c]*Department of Mathematics, Diponegoro University, Semarang 50275, Indonesia*

**Corresponding author Email: tresno.math@undip.ac.id*

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In this paper, we solve an integrated dynamic supplier selection problem and inventory control of a single product inventory system. The deterministic dynamic programming method is used to determine optimal strategies i.e. the optimal supplier and the optimal product volume purchased from each supplier so that the inventory level will follow a reference trajectory as close as possible with minimal cost. A computational experiment is performed to evaluate the problem and its optimal strategies. From the results, the optimal supplier was obtained for each time period, the inventory level was sufficiently closed to the reference level, he optimal product volume from each supplier was determined and the inventory level was followed the given reference trajectory.

Keywords: Deterministic Dynamic programming, Dynamic supplier selection, Inventory control.

1. Introduction

Suppliers, procurement, operations, distribution and customers are the list of the logistics and supply chain management components.[1] Optimization in logistics and supply chain management was developed by many researchers using various methods and approaches. Supplier selection and inventory management are two important components in logistics and supply chain management which contain purchasing cost and holding cost that have to be minimized in order to reduce the spent cost.

Dynamic programming or multi-stage programming can be used to solve many optimization problems which have several characteristics that are the problem can be divided into stages with the decision for each stage is required, each stage has several associate states, decision at a stage describe how the state at current stage is transformed into the state for the next stage, it has principle of optimality and there is recursion function for its stages [12]. Dynamic programming can be divided into two classes which are deterministic where all of information are known with certainty, and stochastic where at least one parameter is uncertain/random. Stochastic dynamic programming can be solved by generating the scenarios of the problem and working forward/backward iteration. Several optimization tools can be used to solve a stochastic dynamic programming by generating the scenario tree. In this paper, we use LINGO to solve the stochastic dynamic optimization **Error! Reference source not found.**

In this paper, a deterministic dynamic programming is used to solve an integrated dynamic supplier selection problem and inventory control for single product inventory system. The optimal strategies that will be determined are the optimal supplier and the optimal product volume so that the inventory level will controlled so that it will follow a reference trajectory as close as possible with minimal cost. A computational experiment will be performed to evaluate the problem and its optimal strategies.

2. Mathematical Model Formulation

Suppose that a retailer (or manufacturer) has a plan to purchase an amount of a product from *S* suppliers for several future time periods which can every day, week, month, etc. The retailer has to satisfy his demand of the product for each time period where the amount of the demand is known with certainty. Let at time period *t*, the retailer has I_t units of the product on his storage/inventory, the demand for time period *t* can be satisfied by the product purchased at *t* or before *t* and to satisfy the demand for the next period can be satisfy by the product purchased at the corresponding period or it can be satisfy by (if possible) the product in the inventory where the retailer is charged for holding cost. The retailer decides to control the inventory level so that it will be near as close as possible to a reference level with minimal cost. Let $X_{t,s}$ denotes the amount of the product purchased from supplier *s* at time period *t*, $U_{t,s}$ denotes the purchasing cost per unit product from supplier *s* at time period *t*, H_t denotes the holding cost per unit product at time period *t* while the maximum capacity of the storage is *M* and D_t denotes the unit product demand. The cost components that will be minimized consisting of the purchasing and the holding cost. As the terminal cost for inventory reference control purposes is defined as $(I_t - r_t)^2$ where r_t denotes the reference level at time period *t*. We formulate the following quadratic integer optimization to solve the problem:

$$\min \sum_{t=1}^{T}\sum_{s=1}^{S} U_{t,s}X_{t,s} + \sum_{t=1}^{T} H_t I_t + \sum_{t=1}^{T}(I_t - r_t)^2 \tag{1}$$

subject to:

$$I_{t-1} + \sum_{s=1}^{S} X_{t,s} - I_t \ge D_t, \forall t \in T; \tag{2}$$

$$X_{t,s} \le C_s, \forall t \in T, \forall s \in S; \quad (3)$$

$$I_t \le M, \forall t \in T; \quad (4)$$

$$X_{t,s}, I_t \in \{0,1,2,...\}, \forall t \in T, \forall s \in S. \quad (5)$$

The objective function (1) consisting of the purchasing cost of all product for all time period, the holding cost for all time period and the terminal cost for inventory reference control purposes. Constraints (2) ensures the demand satisfying of the product for each time period, (3) defines the amount of the product purchased from supplier *s* is not exceeded the supplier capacity, (4) ensures the inventory level is no more than the maximum capacity of the storage, and finally (5) defines the amount of the purchased and stored product is integer.

3. Results and Discussion

For the first numerical experiment, assume that the probability distribution of the random variables are discrete. Suppose that there are two suppliers namely s_1 and s_2. We will solve this problem for 3 periods. Suppose that the purchasing cost from supplier s_1 is \$12 per unit for each of time periods 1, 2 and 3. For time period-1, the purchasing cost from supplier s_2 is \$10 per unit, but for each of time periods 2 and 3, the purchasing cost is random with the probability distribution given by Table (1).

Table 1. Probability distribution of purchase cost from s_2 for time period 2 and 3

Probability	Purchase cost (\$/unit/period)
0.5	10
0.5	12

The supplier capacity for each of supplier s_1 and s_2 is 200 units per period. Let the initial inventory level is 0 item, the holding cost of the product is \$1/unit/period, the warehouse's maximum capacity is 200 units/period and the reference inventory level is 100 units for all time period. The demand for period 1 is known that is 100 units and the demand for each of periods 2 and 3 is random with the probability distribution given by Table (2).

Table 2. Demand's probability distribution for period 2 and 3

Probability	Demand (unit/period)
0.5	100
0.3	120
0.2	150

The decision maker desires that the inventory/stock level must be located at the reference level as close as possible with minimal cost. We solved the optimization problem

Error! Reference source not found. in Windows 8 AMD A6 2.7GHz and 2GB of Memory by using software LINGO 15.0 where the stochastic model class is multi-stage stochastic and the model class is MIQP (mixed integer quadratic programming). This stochastic dynamic program has 36 scenarios as shown by Table (3).

The initial solution i.e. optimal strategy for time period 1 is 0 unit purchased from s_1 and 199 units purchased from s_2. For time period 2, the optimal strategy can be decided after the event of the random variables in time period 2 is revealed. Finally, the optimal strategy for time period 3 can be decided after the event of the random variables in time period 3 is revealed. From the result, the total expected cost is $4819 obtained by the probability and the total cost for all scenarios, that is

$$0.0625 \cdot 4270 + 0.0625 \cdot 4490 + 0.0375 \cdot 4470 + 0.0375 \cdot 4700 + \cdots + 0.0100 \cdot 5862 = 4819.$$

To illustrate how the optimal strategy for each time period is decided, let the demand for period 2 is 100 units and purchasing cost for s_2 for time period 2 is $10/unit, then we have to purchase 0 unit from s_1 and 100 units from s_2 and inventory level for period 2 is 99 units. Let the demand for period 3 is 120 units and purchasing cost for s_2 is $12/unit, then we have to purchase 115 units from s_1 and purchase 0 unit from s_2. This solution is illustrated by Table (4).

Table 4. Illustration of the solution

From Table 4, it can be seen that at time each period 1 and 2, s_2 is the optimal supplier and at time period 3, s_1 is the optimal supplier. It can also be seen that the inventory level is closed to the reference.

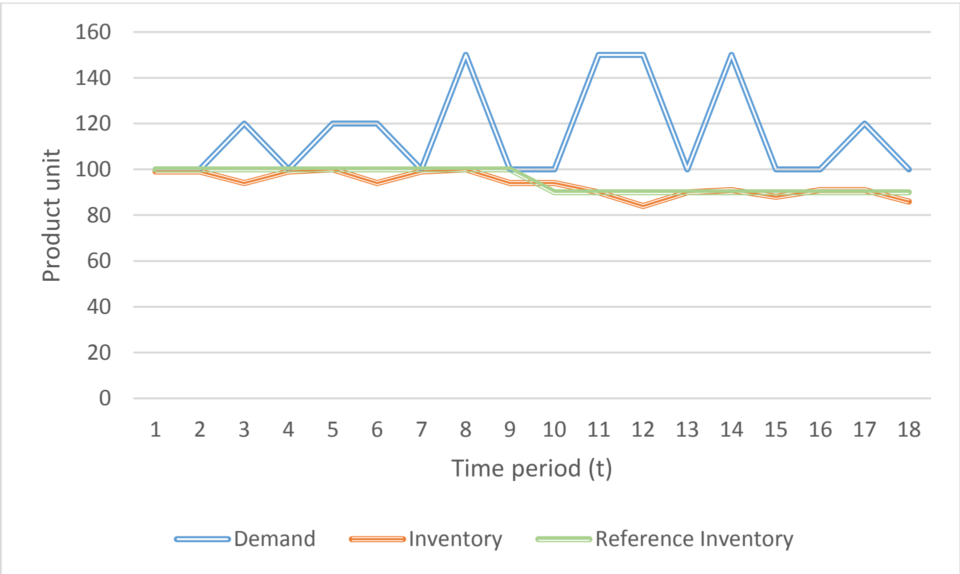

Figure 2. The evolution of the demand, inventory and reference inventory

To evaluate and simulate the evolution of the inventory level with several reference point, we simulate the model for 18 time periods showed by Figure (2). It can be seen that the inventor/stock level of the product is followed the reference point.

From Figure (3), it can be seen that if the demand's standard deviation increases then the total expected cost is also increases. This is caused by the increasing of the demand uncertainty or the demand value has a wide range. If the demand's standard deviation more than 30, we predict that the total expected cost will be increased.

4. Conclusion

In this paper, the mathematical model to determine the optimal strategy to control a single product inventory system integrated with supplier selection in unknown environment was considered. The mathematical model was formulated in a stochastic dynamic optimization and the optimal strategy was determined by using stochastic dynamic programming. Numerical experiments were considered with two cases which are discrete and continue probability distribution. From the results, it can be concluded that the supplier selection problem was solved and the inventory/stock level followed the reference point.

In the next works, we will develop the mathematical model for multi-product case along with order cost, penalty cost for defect or late delivery, service level, etc. Afterwards the random variable can be extended so that other parameters are random. In other works, we will develop the mathematical model so that it can also solve carrier selection problem.

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