Robust Predictive Control Application and Simulation of Inventory Controlling with Imperfect Delivery Process

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***Abstract*— This article is about how optimal decision can be calculated by applying the robust predictive control approach for inventory control problem with imperfect delivery process. The optimal decision is the number of products that need to be ordered in order to satisfy the demand in a case where the delivery process is imperfect. The term "imperfect" in this instance is used to mean a delivery system in which some of the products will become defective along the process or even by the buyer who is the inventory system's decision maker. The amount of the damaged product is uncertain, and is thought of as noise in the state space model. We have formulated a dynamic system for the given inventory control problem as a linear system, with the amount of the ordered product as input, the demand value and the damaged/defected product as measured disturbances in the state equation, and the inventory level as the output. To obtain the optimal input, the robust predictive controller was used to obtain the optimal input, i.e. the product amount ordered at each time. The problem was simulated and the dynamics of the inputs and outputs were observed. From the observations, the optimal number of products ordered was obtained, and the output response, which is the inventory level, was as desired.**

Keywords*— imperfect delivery; inventory problem; robust MPC; supply chain management

I. PRELIMINARY

Logistics & supply chain contains many components from the upstream (e.g. raw material supplier) to downstream (e.g. end user). One of the essential components is the inventory system, which contributes to the operational costs including procurement and storage. Hence, the inventory system should be optimized in order to reduce operational cost and gain profit. To optimize the system, the mathematical model approach is a good choice, and it involves the dynamics of the inventory level being modelled as a dynamic system. One of the pioneer inventory models is available in a linear dynamic system with a linear quadratic control method was applied [1], while a quite advanced model was formulated with the multi-product inventory system and its predictive controller [2]. This more advanced classic model was developed in the process of inventing a multi-supplier system [3]. The above mentioned references were used a control method in the process of calculating the optimal decision, even though other approaches were also developed by other researchers, like the optimal policy approach [4] and particle swarm optimization approach [5]. Apart from these, many application types of research have also been conducted, some of such examples include the application discussion on the pharmaceutical industry [6], hospital inventory management, [7] and emission control [8]. Some more complex problems, like uncertain demand, were solved using existing works [9], joint model with working environment [10], modification with non-linear holding cost [11], joint model with pricing in service [12], a model with returns and sudden obsolescence [13] and a model considering stochastic price process [14].

The authors thank to Universitas Diponegoro for funding support under RPI 2019 research grant contract No. 474-89/UN7.P4.3/PP/2019.

However, one of the most impressive modern control methods is the predictive control method (MPC). This is an optimization-based control method [15], which is not only great for a simple dynamic system, but it is also applicable for complex dynamical systems like hybrid system [16], [17]. MPC works by using the prediction vector along the prediction horizon and an optimization method for minimizing some objective function in order to calculate the optimal input (decision). Many research articles are applying the MPC control method, which describes the performance of the MPC. For example, in industrial manufacturer applications, MPC was applied to solve supplier selection problem [18] and milling process optimization [19], whereas in more general control problem field MPC was applied effectively to control some instrumentations like electromagnetic tethered satellite system [20], unmanned aerial vehicle [21], and many more.

In this paper, a dynamic system was formulated to describe the dynamics of an inventory system with imperfect delivery process. The robust MPC method was used to calculate the optimal output, that is, the amount of products that is needed to be purchased by the decision maker at each time. Also, some computational experiments were performed to simulate the discussed inventory system and its input-output responses.

II. MATERIALS AND METHODS

A. Problem Definition

An inventory or warehouse system which is containing single supplier and single product, will be controlled in a such that the number of stored products, that is the inventory level, would follow some reference point decided by the decision maker (DM). The DM is to know the amount of the product which was demanded, and this must be met by the inventory system.

Normally, the amount of products sent by the supplier should be equal to the amount received on delivery. However, in this case where the delivery is imperfect, during the delivery process from the supplier to the warehouse, some products were damaged and the amount of products reaching the warehouse would be less than the amount that was supplied. Now, the problem is how to determine the optimal product amount that should be purchased by the DM in order for the stock to satisfy the demand.

B. Assumptions

To formulate the mathematical model of the problem, we apply some assumptions below:

1. There is no expiration on the product along the optimization period;
2. The supplier meets all of the purchased products ordered.

C. Mathematical Model

Let $y(k) \in \mathbb{R}^+$, denote the inventory level or stored product volume at time $k \in \mathbb{Z}^+$ with an initial value $y(0) = y_0$. Let $d(k) \in \mathbb{R}^+$ denote the demand value at time k. Let $u(k) \in \mathbb{R}^+$ denote the ordered product volume from the supplier at a time k. The product ordered at time k will be received at time $(k+l)$ where $l \in \{0,1,2,...\}$ is lead time delay (period of time from ordering to receiving the product). The imperfect delivery process in this case is presented as the damaging of the products during the delivery process. However, the damaged products will be rejected and not entered into inventory. If the rejected product volume is denoted u_d which is uncertain and assumed to be a random variable with some known probability distribution, then the stored product volume in the warehouse at any $k > 0$ can be formulated as follows:

$$\begin{aligned} y(k) = u(-l) - u_d + u(1-l) - u_d + u(2-l) - u_d + \ldots \\ + u(k-1-l) - u_d - d(0) - d(1) - d(2) - \ldots - d(k-1). \end{aligned} \tag{1}$$

Let $l = n+1$, $x_1(k) = y(k)$ and $x_i(k) = u(k - n_p + i)$, $\forall i = 2,\ldots,n$, then (1) can be rewritten as

$$\begin{cases} x_1(k+1) = x_1(k) + x_2(k) - d(k) \\ x_2(k+1) = x_3(k) \\ \cdots \\ x_{n-1}(k+1) = x_n(k) \\ x_n(k+1) = u(k) - u_d \\ y(k) = x_1(k). \end{cases} \tag{2}$$

Let $x(k) = [x_0(k), x_1(k), x_2(k), \ \ldots \ , x_n(k)]^T$, then (2) can be rewritten as follows:

$$x(k+1) = \begin{pmatrix} x_1(k+1) \\ x_2(k+1) \\ \vdots \\ x_{n-1}(k+1) \\ x_n(k+1) \end{pmatrix} = \begin{pmatrix} x_1(k) + x_2(k) - d(k) \\ x_3(k) \\ \vdots \\ x_n(k) \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ \vdots \\ x_n(k) \end{pmatrix} + \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \\ \vdots & \ddots & \vdots \\ 0 & 0 & 0 \\ 1 & 0 & -1 \end{pmatrix} \begin{bmatrix} u(k) \\ d(k) \\ u_d \end{bmatrix} \tag{3}$$

$$y(k) = (1,0,...,0)\, x(k).$$

Alternatively, in a linear state space, it can be rewritten as follows:

$$\begin{cases} x(k+1) = Ax(k) + B\overline{u}(k) \\ \quad y(k) = Cx(k) \end{cases} \tag{4}$$

where

$$x(k) = \begin{pmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ \vdots \\ x_n(k) \end{pmatrix}, \overline{u}(k) = \begin{bmatrix} u(k) \\ d(k) \\ u_d \end{bmatrix}, A = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$B = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \\ \vdots & \ddots & \vdots \\ 0 & 0 & 0 \\ 1 & 0 & -1 \end{pmatrix}, C = (1,0,...,0).$$

D. Robust MPC (Revisiting)

We are revisiting the robust MPC method from [1]. Consider a linear time-invariant system with disturbances modeled as a system

$$\begin{aligned} x(k+1) = Ax(k) + B_1 u(k) + B_2 u_d(k) \\ y(k) = Cx(k) \end{aligned} \tag{5}$$

where x is the state vector, u is the input vector, y is the output vector, and u_d is the disturbance vector assumed to be a white noise with $E(u_d u_d') \geq 0$ as its covariance matrix. Let $\hat{x}$ denotes the optimal filtered state estimated by using Kalman filter, then we have [12]

$$\begin{aligned} \hat{x}(k+1) = A\hat{x}(k) + Bu(k) \\ \hat{x}(k+m) = A\hat{x}(k+m-1) + B\hat{u}(k+m-1) \text{ for } m > 1. \end{aligned} \tag{6}$$

This estimated state is derived using the prediction horizon, and then MPC is used to minimize the objective function as the control goal (reference tracking purposes) as follows:

$$J(k) = \sum_{i=0}^{N_y} \left\| \left(y(k+i) - y_r \right) \right\|_Q^2 + \sum_{i=0}^{N_u} \left\| u(k+i) \right\|_R^2 \tag{7}$$

where $Q = diag\{Q_i\}_{i=0,1,\ldots,N_y}$ and $R = diag\{R_i\}_{i=0,1,\ldots,N_u}$ are weighting matrices which are semi-definite positive. Finally, the optimal input is achieved by minimizing this objective function using quadratic programming where the robustness of the solution of (7) was discussed in [15].

III. Computational Simulation

Consider system (4) with l = 3. Then it can be rewritten as follows:

$$x(k+1) = \begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \\ x_4(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ x_4(k) \end{bmatrix} + \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} u(k) \\ d(k) \\ u_d \end{bmatrix} \quad (8)$$

$$y(k) = [1,0,0,0]\, x(k).$$

The value of parameters in the model are summarized in TABLE 1.

TABLE 1. The parameter value for simulation

Parameter	Value	Note
Q	I	Identity matrix
R	I	Identity matrix
y_r	80	Assumed to be constant for all time periods
N_y	10	Horizon output prediction
N_u	10	Horizon input prediction

MATLAB R2018b was used to perform the simulation in a daily used personal computer with Windows 10 operating system, 6 GB of memory and 2.3 GHz of Processor. The results can be seen in Figures 1 – 2.

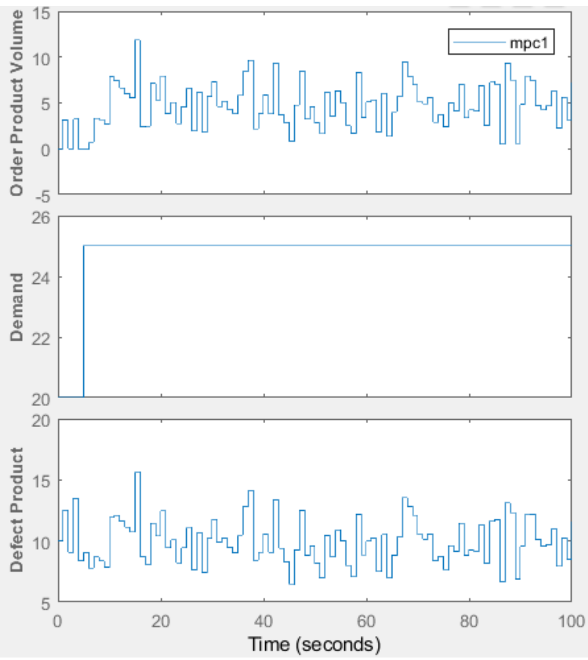

Fig. 1. Optimal Order product volume, demand value, and defect product volume

Figure 1 shows the product volume to be ordered, demand value, and also the defective product units. The ordered product volume generated by the RMPC method shows the optimal product amount that needs to be ordered each time. In this simulation, the demand value for each time is in Fig. 1. The defective products units shown in Fig. 1 was generated as normally distributed with a mean of 10 and a variance of 5. If these values were to be applied in dynamic system, the expected responses, that is the inventory level, are shown in Fig. 2.

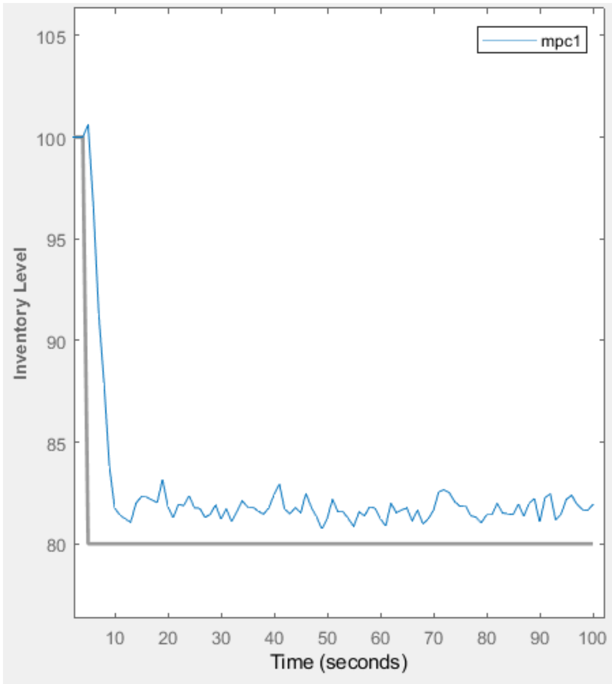

Fig. 2. The inventory level response

Figure 2 shows us the inventory level over the optimization period. Initially, the inventory level is 100 units, with the reference/set point being 80 units, and it can be seen that the amount of stored products is close to the set point. In this simulation, the reference inventory level is constant for all periods in order to observe how fast the actual stock level of the product reaches the reference point. From Fig. 2, it can be seen that the inventory level reached the set point in time. However, in a real-life application, the reference point can vary for each time, even though the controller will probably need several periods to bring the inventory level to its reference point.

IV. Conclusions

In this paper, the RMPC was used to calculate the optimal decision for a single product in a case where the delivery system is an imperfect one with a dynamic model of the product stock level being a linear system. With the simulation, the optimal product decision was derived and the actual product stock level was observed to be sufficiently close to the desired reference level.

For further research, the results gotten by using this method can be compared with other methods in a bid to find the best method to use for inventory control. Also, a case study using actual data from a real-life situation can be considered in order to analyze the performance of the controller.

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