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Banking Loan Dynamics with Dividen Payments

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Abstract

A difference equation of banking loans with dividend payments is constructed, with the future distribution of loans based on the bank's rational expectation, which is the current marginal profit. The equilibrium of the model is calculated and analyzed. The equilibrium is found to be stable if the dividend payment parameter lies between the values for the flip and transcritical bifurcations. Too much dividend payment will render the loan null, while too little dividend payment will render the loan periodic or even chaotic. The effect of dividend payment in conjunction with other parameters on loan stability is also investigated.
Keywords: bifurcations, chaos, difference equation, dividend payment, loan dynamics
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1. INTRODUCTION

Banking dynamics is one of the financial issues in the banking industry that merits analysis in order to study their behavior or obtain data for future regulation planning. We may study them using economics, finance, econometrics, statistics, or mathematics.

Numerous mathematical tools have been employed by researchers to examine banking dynamics, such as constrained optimization of profit to examine the behavior of monopolistic or oligopolistic banking institutions in determining their optimal portfolio [22, 24, 18], random graph theory or stochastic systems to simulate the finance system's dynamics [23, 20, 21, 10], dynamical systems of bank balance sheet variables to examine their dynamics over time [25, 8, 7, 9], and difference equations to examine the dynamics of bank loan issuance incorporating numerous banking policies [17, 13, 14, 5, 6] or banking costs [4, 3].

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In this study, we employ a difference equation approach to examine the dynamics of bank loans with dividend payments. In the finance industry, dividend payments play a significant role as a crucial mechanism for distributing profits to shareholders. According to research [16], dividend-paying banks are regarded favorably by investors, attracting a broader investor base and possibly increasing the stock's valuation. On the other hand, a decrease or cessation of dividend payments may be seen as an indication of financial distress or a move to conserve cash during difficult economic times [2]. We follow the model in [6, 11] by using four bank balance components, such as deposit, equity, reserve requirement, and loan, to study the effects of dividen payment on the stability of loan.

2. DYNAMIC MODEL OF LOAN

A bank's balance sheet could include the following items: deposits (D), equity (E), reserve requirement (R), and loans (L). Thus, we have the identity of the balance sheet,

$$L + R = D + E.$$

We are able to write since a percentage of the deposit goes toward the reserve requirement,

$$R = \rho D, \quad 0 < \rho < 1.$$

We assume that the ρ parameter is set to the minimum portion value determined by the central bank.

The equity in the bank will be covered by the capital standards set by the central bank. According to data from the banking sector, as demonstrated in [5], the equity to loan ratio is likely to stay the same in practice. This enables us to write,

$$E = \kappa L, \quad 0 < \kappa < 1.$$

The deposit serves as the balance variable in this model,

$$D = L + R - E = L + \rho D - \kappa L, \quad \text{or} \quad D = \frac{1-\kappa}{1-\rho} L.$$

Consider that the model's time has discrete values, $t = 0, 1, 2, \ldots$. The bank's loan distribution for the subsequent period is defined by the loan's marginal profit, in accordance with the gradient adjustment process [12, 17]. The model is shown below,

$$L_{t+1} = L_t + \alpha_L L_t \frac{\partial \pi_t}{\partial L_t}, \tag{1}$$

where α_L is known as the adjustment speed parameter.

The bank's profit is calculated by subtracting the loan interest ($r_L L$) from the deposit expense ($r_D D$), the dividen payment (δE), and the operating costs of the bank (C). Here, the parameter δ is assumed to the portion of dividen payments. In this paper, we study the parameter δ, what is its role in banking loan dynamics.

Using the assumptions of the Monti-Klein model [22, 24], the loan and deposit interest rates are defined as $r_L = a_L - b_L L$ and $r_D = a_D + b_D D$, where $a_L, b_L, a_D, b_D > 0$. The bank operating cost is defined by $C = c_D D + cDL + c_L L$, where $0 < c_D, c, c_L < 1$. Here, we assume that in the bank operating cost there is a diseconomies of scope [15].

The profit at time t is computed by

$$\begin{aligned}\pi_t &= r_L L_t - r_D D_t - \delta E_t - C_t \\ &= \left(a_L - \left[\delta\kappa + c_L + (a_D + c_D)\left(\frac{1-\kappa}{1-\rho}\right)\right]\right) L_t - \left[b_L + c\left(\frac{1-\kappa}{1-\rho}\right) + b_D\left(\frac{1-\kappa}{1-\rho}\right)^2\right] L_t^2.\end{aligned}$$

Next, we determine the marginal loan profit as follows,

$$\frac{\partial \pi_t}{\partial L_t} = a_L - \left[\delta\kappa + c_L + (a_D + c_D)\left(\frac{1-\kappa}{1-\rho}\right)\right] - 2\left[b_L + c\left(\frac{1-\kappa}{1-\rho}\right) + b_D\left(\frac{1-\kappa}{1-\rho}\right)^2\right] L_t. \tag{2}$$

Substituting (2) for (1) yields

$$\begin{aligned}L_{t+1} = L_t + \alpha_L L_t \Bigg(a_L - \left[\delta\kappa + c_L + (a_D + c_D)\left(\frac{1-\kappa}{1-\rho}\right)\right] \\ -2\left[b_L + c\left(\frac{1-\kappa}{1-\rho}\right) + b_D\left(\frac{1-\kappa}{1-\rho}\right)^2\right] L_t\Bigg).\end{aligned} \tag{3}$$

3. STABILITY AND BIFURCATION ANALYSIS

3.1. Local stability analysis

By putting $L_{t+1} = L_t$ and solving for L, we may get the equilibrium from the difference equation (3). There are two equilibrium points: the zero loan equilibrium ($L^*_{(0)} = 0$) and the non-zero loan equilibrium

$$L^*_{(p)} = \frac{a_L - \left[\delta\kappa + c_L + (a_D + c_D)\left(\frac{1-\kappa}{1-\rho}\right)\right]}{2\left[b_L + c\left(\frac{1-\kappa}{1-\rho}\right) + b_D\left(\frac{1-\kappa}{1-\rho}\right)^2\right]}.$$

Positive values of L^*_p are required for economic meanings, or in other terms

$$0 < \delta < \frac{a_L - \left(c_L + (a_D + c_D)\left(\frac{1-\kappa}{1-\rho}\right)\right)}{\kappa} \tag{4}$$

with condition $a_L > c_L + (a_D + c_D)\left(\frac{1-\kappa}{1-\rho}\right)$.

Let's say that $L_{t+1} = f(L_t)$ is used to rewrite the difference equation (3). The one-dimensional map (3) is stable, according to [1], if

$$|f'(L^*)| < 1.$$

The stability of the loan equilibrium values $L^*_{(0)}$ and $L^*_{(p)}$ is established by the theorem below.

Theorem 1. *The loan equilibium $L^*_{(0)}$ is unstable. Meanwhile, the loan equilibrium $L^*_{(p)}$ is stable if*

$$\delta > \frac{a_L - \left(c_L + (a_D + c_D)\left(\frac{1-\kappa}{1-\rho}\right)\right)}{\kappa} - \frac{2}{\kappa\alpha_L}.$$

Proof. First, we calculate the first derivative of f,

$$f'(L_t) = 1 + \alpha_L\left(a_L - \left[\delta\kappa + c_L + (a_D + c_D)\left(\frac{1-\kappa}{1-\rho}\right)\right]\right)$$
$$-4\alpha_L\left[b_L + c\left(\frac{1-\kappa}{1-\rho}\right) + b_D\left(\frac{1-\kappa}{1-\rho}\right)^2\right]L_t.$$

For the zero loan equilibrium, we have

$$f'\left(L^*_{(0)}\right) = 1 + \alpha_L\left(a_L - \left[\delta\kappa + c_L + (a_D + c_D)\left(\frac{1-\kappa}{1-\rho}\right)\right]\right) > 1.$$

Therefore, $L^*_{(0)}$ is unstable.
For the positive loan equilibrium, we have

$$f'(L^*_{(p)}) = 1 - \alpha_L\left(a_L - \left[\delta\kappa + c_L + (a_D + c_D)\left(\frac{1-\kappa}{1-\rho}\right)\right]\right). \tag{5}$$

From Eq. (5), it is clear that $f'(L^*_{(p)}) < 1$. On the other hand, $f'(L^*_{(p)}) > -1$ is satisfied if

$$\delta > \frac{a_L - \left(c_L + (a_D + c_D)\left(\frac{1-\kappa}{1-\rho}\right)\right)}{\kappa} - \frac{2}{\kappa\alpha_L}$$

Thus, $L^*_{(p)}$ is stable if $\delta > \frac{a_L - \left(c_L + (a_D + c_D)\left(\frac{1-\kappa}{1-\rho}\right)\right)}{\kappa} - \frac{2}{\kappa\alpha_L}$. □

3.2. Bifurcations

We follow the Jury stability requirement for a one-dimensional map as stated in [19]. The conditions $f'(L^*_{(p)}) = -1$ and $f'(L^*_{(p)}) = 1$ respectively imply that the loan equilibrium will lose stability owing to flip or period-doubling bifurcation, and transcritical bifurcation. This study's goal is to ascertain how the dividen payment parameter δ affects the stability of loans in the banks balance sheet. Therefore, the parameter will function as the bifurcation parameter. The following theorem is arrived at through a sequence of simple calculations.

Theorem 2. *The positive loan equilibrium $L^*_{(p)}$ may lose its stability due to transcritical bifurcation when $\delta = \delta^T$, where*

$$\delta^T = \frac{a_L - \left(c_L + (a_D + c_D)\left(\frac{1-\kappa}{1-\rho}\right)\right)}{\kappa}.$$

On the other hand, $L^*_{(p)}$ due to flip bifurcation when $\delta = \delta^F$, where

$$\delta^F = \frac{a_L - \left(c_L + (a_D + c_D)\left(\frac{1-\kappa}{1-\rho}\right)\right)}{\kappa} - \frac{2}{\kappa\alpha_L}.$$

Proof. The transcritical bifurcation value is obtained directly by solving the following equation

$$f'(L^*_{(p)}) = 1 - \alpha_L\left(a_L - \left[\delta\kappa + c_L + (a_D + c_D)\left(\frac{1-\kappa}{1-\rho}\right)\right]\right) = 1$$

for parameter δ, which resulting $\delta = \frac{a_L - \left(c_L + (a_D+c_D)\left[\frac{1-\kappa}{1-\rho}\right]\right)}{\kappa}$. Similarly, by solving

$$f'(L^*_{(p)}) = 1 - \alpha_L\left(a_L - \left[\delta\kappa + c_L + (a_D + c_D)\left(\frac{1-\kappa}{1-\rho}\right)\right]\right) = -1$$

for parameter δ, the flip bifurcation value is obtained as $\delta = \frac{a_L - \left(c_L + (a_D+c_D)\left(\frac{1-\kappa}{1-\rho}\right)\right)}{\kappa} - \frac{2}{\kappa\alpha_L}$. □

It is simple to verify that $\delta^F < \delta^T$ from Theorem 2. According to Theorem 1, $L^*_{(p)}$ is stable if $\delta > \delta^F$. As a result, we have upper and lower bounds for the parameter δ to ensure the stability of the loan equilibrium that is $\delta^F < \delta < \delta^T$.

We have that $0 < \delta < 1$, so in order to have economic meanings, both transcritical and flip bifurcations value must be between 0 and 1. Note that,

$$\begin{aligned} 0 < \delta^T < 1 &\Leftrightarrow 0 < \frac{a_L - \left(c_L + (a_D + c_D)\left(\frac{1-\kappa}{1-\rho}\right)\right)}{\kappa} < 1 \\ &\Leftrightarrow c_L + (a_D + c_D)\left(\frac{1-\kappa}{1-\rho}\right) < a_L < \kappa + c_L + (a_D + c_D)\left(\frac{1-\kappa}{1-\rho}\right) \end{aligned}$$

The last expression is always true, because we have the positivity condition in (4). In the meantime, for the case of flip bifurcation value, we have

$$0 < \delta^F < 1 \quad \Leftrightarrow \quad 0 < \frac{a_L - \left(c_L + (a_D + c_D)\left(\frac{1-\kappa}{1-\rho}\right)\right)}{\kappa} - \frac{2}{\kappa \alpha_L} < 1$$

$$\Leftrightarrow \quad \frac{2}{\alpha_L} + \left(c_L + (a_D + c_D)\left[\frac{1-\kappa}{1-\rho}\right]\right)$$

$$< a_L < \kappa + \frac{2}{\alpha_L} + \left(c_L + (a_D + c_D)\left[\frac{1-\kappa}{1-\rho}\right]\right)$$

Therefore, we have the following corollary.

Corollary 1. *The transcritical bifurcation value δ^T will exist if*

$$a_L < \kappa + c_L + (a_D + c_D)\left(\frac{1-\kappa}{1-\rho}\right)$$

while for the flip bifurcation δ^F will exist while

$$a_L < \kappa + c_L + (a_D + c_D)\left(\frac{1-\kappa}{1-\rho}\right) + \frac{1}{\alpha_L}$$

4. NUMERICAL EXPERIMENTS

4.1. Simulation

The results of the previous section are illustrated and validated using several numerical simulations. The parameter values listed in Table 1 are used to run the simulations. Although the values of these parameters were chosen primarily for simulation purposes, they nonetheless satisfy the loan interest rate parameter a_L criteria in Corollary 1 and the positive condition of loan equilibrium in (4) The numbers from Table 1 can be used to determine the bifurcation values of the dividen payment parameter. One of the values is $\delta^F = 0.2702$ and $\delta^T = 0.9841$ for the α_L value of 32.

We examine how the loan L_t trajectory has affect over time in response to variations in the benchmark dividen payment parameter δ. Graph presented in Figure 1 depict convergent loan trajectory data. Note that the larger the value of δ the smaller the value of loan L_t, and if δ is quite small, it makes the loan trajectory becomes fluctuative.

Bifurcation diagrams are a useful tool for studying a map's dynamics because they show in great detail when a map is stable or unstable and when it can act chaotically. For the dividen payment parameter δ, we provide a bifurcation diagram in Figure 2a.

Table 1: The value of parameters for simulation.

Parameter	Description	Value
ρ	Minimum reserve requirement	0.12
a_L	Parameter of loan interest rate	0.097
b_L	Parameter of loan interest rate	0.05
a_D	Parameter of deposit interest rate	0.01
b_D	Parameter of deposit interest rate	0.05
κ	equity to loan ratio	0.05
δ	dividen payment parameter	vary
c_D	Marginal cost of deposit	0.05
c_L	Marginal cost of loan	0.05
α_L	Speed of adjustment for channelling loan	30,31, and 32

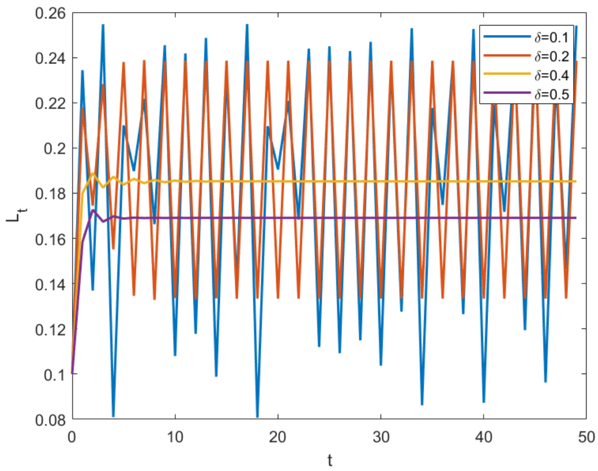

Figure 1: Graphs of loan L_t versus time t for various values of dividen payment parameter δ.

The graph shows that the loan equilibrium is zero when the transcritical bifurcation value, $\delta > \delta^T$, surpasses the dividen payment. We get a positive and stable loan equilibrium when the dividen payment is between the transcritical and flip bifurcation values, $\delta^F < \delta < \delta^T$. Lowering the dividen payment raises the lending equilibrium within this range. The loan equilibrium behaves periodically starting from $\delta < \delta^F$ to lower value and even results in chaos.

The graph of the Lyapunov exponent, which is related to 2a, is shown in Figure 2b. The graph of the Lyapunov exponent is depicted with black dots if it does not exceed zero and red dots if it does. A positive Lyapunov exponent indicates that the dynamics of the loan have become chaotic.

Additionally, by adjusting the value of α_L, which is 30, 31, and 32, we show three bifurcation diagrams, namely Figures 2a, 2c, and 2e. As can be shown, the flip bifurcation is affected by α_L, but not the transcritical bifurcation. This is consistent with the conclusions we reached and are reported in Theorem 2.

It is always fascinating to see how a map behaves when its dynamics take on an unstable, chaotic, or even periodic, state. Its cobweb diagram is one of its depictions of a map. We can observe the cobweb diagram there in real time as it develops until it finds equilibrium. Figure 3a shows the cobweb diagram of L_t for $\delta = 0.25$. Figure 3b shows the cobweb diagram for $\delta = 0.025$. The cobweb diagram's chaotic track is clearly seen in this situation. The black trajectory line repeatedly veers across the blue dotted curve as t increases. Thus, the map is chaos.

We attempt to give an outline of chaotic behavior in order to support the earlier findings. The chaotic loan behavior will be investigated using slightly varied initial values in the simulation that follows. Figure 4 shows an illustration of the chaotic behavior of the loan map for the case $\delta = 0.025$. Two graphs of the loan map with somewhat different initial values are shown in the graphic. Initial values for the blue and red graphs are $L_0 = 0.5$ and $L_0 = 0.50001$, respectively. In the picture, the blue and red graphs first resemble one another clearly before splitting off and taking independent paths.

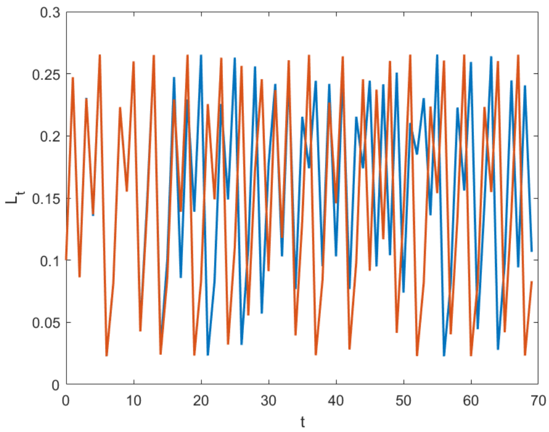

Figure 4: Sensitivity dependence of the chaotic loan dynamics on the initial condition for dividen payment parameter $\delta = 0.3$ The simulation uses $\alpha_L = 32$

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4.2. Sensitivity analysis

It has been demonstrated in Theorem 1 that the stability of the positive loan equilibrium $L^*_{(p)}$ has a requirement. Theorem 1 may be rewritten as: $L^*_{(p)}$ is stable if $S < 1$ by

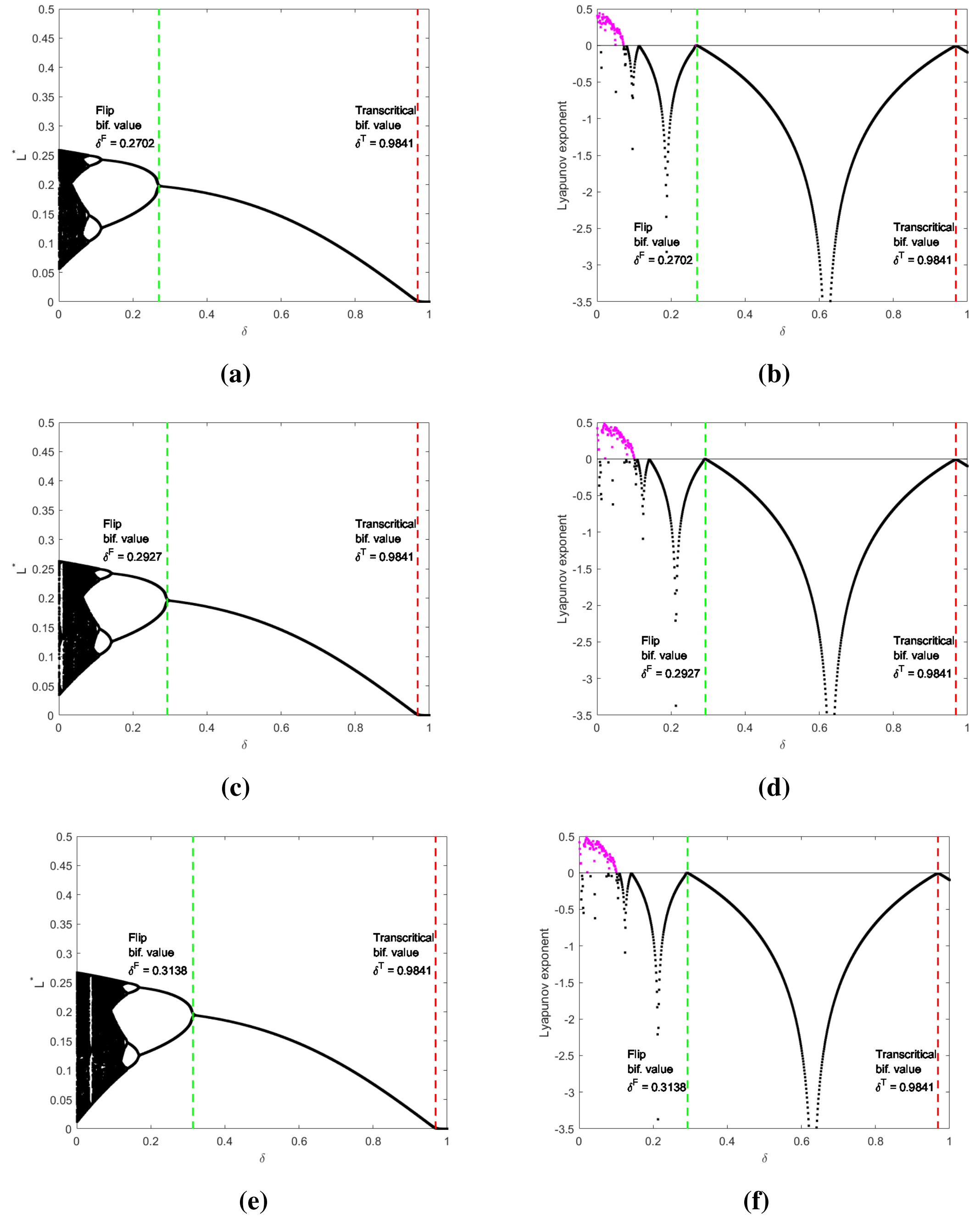
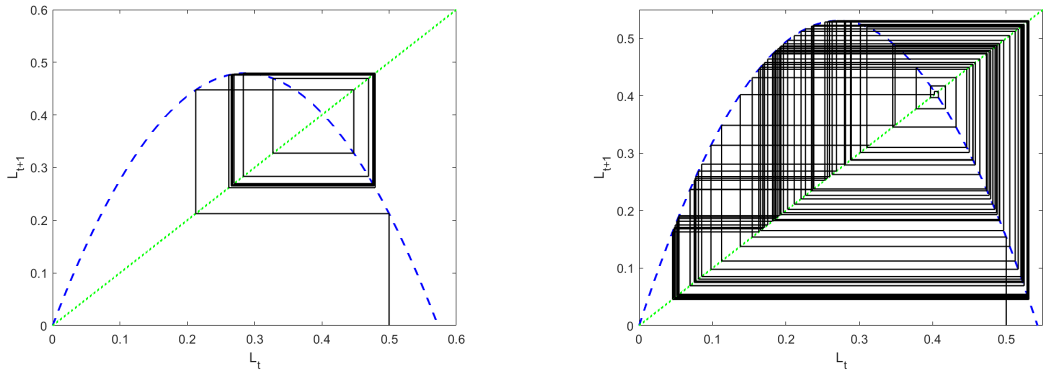


Figure 2: Bifurcation diagrams of the dividen payment parameter δ and the corresponding Lyapunov exponent using different values of α_L, where $\alpha_L = 30$ (up), $\alpha_L = 31$ (middle), and $\alpha_L = 32$ (bottom).

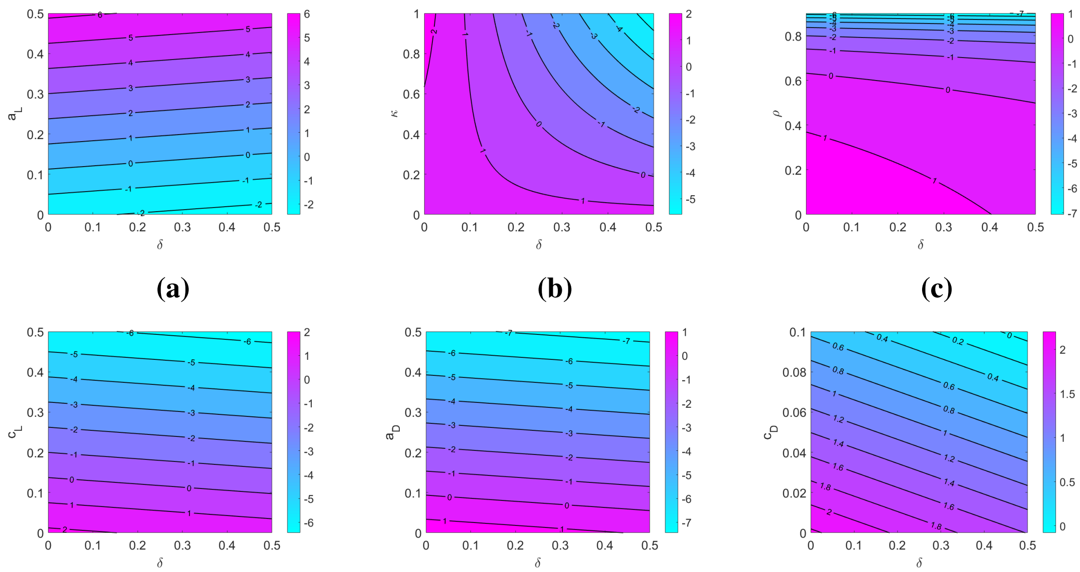
specifying $S = 1 - \alpha_L \left(a_L - \left[\delta\kappa + c_L + (a_D + c_D) \left(\frac{1-\kappa}{1-\rho} \right) \right] \right)$. Since the goal of this paper is to evaluate the dividen payment policy—more specifically,it is interesting to observe the role of combination of parameter δ with other parameters taken from



(a) **(b)**

Figure 3: Panel (a) shows the cobweb diagram loan when the dividen payment parameter $\delta = 0.25$ and panel (b) shows the cobweb diagram loan with chaotic behavior when the dividen payment parameter $\delta = 0.025$.

$Q = \{a_L, \kappa, \rho, c_L, a_D, c_D\}$ in maintaining the stability of loan. We show the contour plot of S in the δq-plane, where $q \in Q$, in Figure 5. By observing the colorbar with a value below one, we may see which portion of the loan equilibrium is stable.



(a) **(b)** **(c)**

(d) **(e)** **(f)**

Figure 5: Contour plot of S in a plane of combination of parameter δ with (a) a_L, (b) κ, (c) ρ, (d) c_L, (e) a_D, (f) c_D. These simulations use $a_L = 0.2$.

The contour plots in Figure 5 can be viewed as sensitivity analysis of a combination of the dividen payment parameter (δ) with other parameters appeared in the banking industry such as loan interest rate parameter (a_L), equity to loan ratio (κ), reserve requirement policy (ρ), marginal cost of loan (c_L), deposit interest rate parameter (a_D), and marginal cost of deposit (c_D), in contributing to the stability of loans. In Figure 5a, take note that the stability of the loan equilibrium may lose its stability as a_L increases; this is illustrated by the point at which, $a_L \approx 0.18$. On the other hand Figures 5c, 5d, 5e, and 5f show that the stability of the loan equilibrium will rise as the three values in a row of the parameters ρ, c_L and a_D increase. In contrast, for the $\kappa - \delta$ combination, the loan equilibrium will become unstable as the κ and δ numbers decrease, this can be seen in Figure 5b.

5. CONCLUSION

This paper analyzes the issue to determine how dividend payments would impact the loan dynamics of a bank. Reducing the dividend payment increases the loan balance. The findings of this study suggest that the dividend payment should not be exceedingly high or low. In the future, if the dividend payment is excessively high, the loan will be annulled. A dividend payment that is insufficient could destabilize the loan and cause turmoil in the interim. In the banking sector, dividend payments play a significant role in the process of transferring earnings to shareholders. Dividends are a source of investor interest, a symbol of financial stability, and they enhance the performance and value of equities in the banking industry. Investors, regulators, and other market participants must comprehend the relevance of dividend payments in the banking sector in order to evaluate the financial health and future prospects of banks.

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