Optimizing Raw Parts Procurement and Inventory Under Excess Demand Values and Unknown Prices in Post Pandemic Situations via Fuzzy Programming

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Abstract – This study uses a fuzzy programming model to optimize part procurement and inventory management under excess demand values and unknown prices. It has been conducted in pandemic and post-pandemic situations for health-care-related companies and non-essential related services, where demand values tend to be surplus and unpredictable. The problem is modeled as mathematical optimization, and the objective function is the expectation value in the fuzzy sense of the total costs considered. The decision is centered on the number of parts for each supplier or those stored in the inventory for future use. Finally, numerical experiments are performed to demonstrate how the problem is solved. The results indicate that the optimal decisions have been derived, and therefore the proposed approach can be adopted by manufacturing/retail/service companies. **Copyright © 2023 Praise Worthy Prize S.r.l. - All rights reserved.**

Keywords: Excess Demand, Fuzzy Programming, Post-Pandemic, Procurement, Supply Chain Management

Nomenclature

- \mathcal{P} Set of part types, $\mathcal{P} = \{1, 2, ..., P\}$
- S Set of suppliers, $S = \{1, 2, ..., S\}$
- \mathcal{B} Set of brands, $\mathcal{B} = \{1, 2, ..., B\}$
- \mathcal{T} Set of observation periods, $\mathcal{T} = \{1, 2, ..., T\}$
- X_{tsp} Amount of part p purchased to supplier s
- $\begin{array}{l} \text{at observation period } t \\ Y_{tb} & \text{Number of on hand brand } b \\ \text{at observation period } t \end{array}$
- Z_{ts} Binary number that is representing whether the supplier *s* is charged for order cost at period *t* will be 1 if yes or 0 if not
- W_s Binary number representing whether supplier s is chosen as a new supplier (1) or not (0)
- S_{ts} Number of trucks that delivers part from supplier *s* in period *t*
- i_{tp}^X Inventory level of part *p* in period *t*
- i_{tb}^{Y} Inventory level of brand b in period t
- $\widetilde{AC}_{tb} \quad \begin{array}{l} \text{Assembly/processing cost of brand } p \\ \text{at observation period } t \end{array}$
- \widetilde{BP}_{tb} Selling price of brand *b* at observation period *t* \widetilde{PP}_{tsp} Price of part *p* at supplier *s*
- at observation period t
- $\widetilde{TC}_{ts} \qquad \text{One truck delivery cost from supplier } s \\ \text{at observation period } t$
- \tilde{O}_s Cost that occurred while ordering parts from supplier *s*

\widetilde{LR}_{tsp}	Late on delivery rate (in percentage) of ordered part from supplier s of part p
\widetilde{DR}_{tsp}^X	Percentage of rejected part p from supplier s
\widetilde{DR}_{tb}^{Y}	Percentage of rejected part p from supplier s
\widetilde{NC}_s	New contract cost for a new supplier at observation period <i>t</i>
\tilde{D}_{tb}	Demand value (unit) of brand b in period t
Rp_{pb}	Number of parts <i>b</i> needed to make one unit brand <i>b</i>
С	Full truckload maximum capacity
SC_{tsp}	Maximum capacity of supplier s to supply part p in period t
PD_{tp}^X	Penalty cost for defected part p
PL_{tp}	Penalty cost for late delivery of part <i>p</i> for each period
PD_{tb}^{Y}	Penalty cost for defected brand b
HP_{tp}	Cost for storing a unit part p per period
HB_{tb}	Cost for storing a unit brand b per period
MP _{tp}	Maximum warehouse capacity to store part p in period t
MB _{tb}	Maximum warehouse capacity to store brand b in period t

I. Introduction

Several pandemics have been experienced globally, and there are possibilities of hitting again in the future. Since 2019, the Covid-19 pandemic has been hitting

different countries. Numerous sectors, including the manufacturing and hospitality industries, are affected by this pandemic, and their future is still not clear. The services of non-essential companies have drastically decreased during the Covid-19 pandemic. These companies have tried to recover their businesses and services towards the end of the pandemic. Therefore, several sectors are predicted to have excess demand, especially in hospitality-related industries. This will raise a situation where the manufacturers or the service providers can have excess demand values and shortages of their products. Another concern is that health-carerelated companies have excess demand values for various products/services. This extraordinary condition can satisfy the actual demand since the production capacity, and the available raw materials/parts are limited. A more complicated problem occurs when competition exists between companies to obtain raw materials and supply manufacturers with better prices. Therefore, the cheapest raw materials/parts will not be available. A decision should be made to deal with the competition optimally, but possibly not with a minimal cost in purchasing raw materials/parts and selling products/services.

Furthermore, a mathematical optimization model can provide an optimal decision under the given situations.

Therefore, a new appropriate model is required to handle this situation. First, the raw parts procurement from suppliers should be settled since some mathematical programming models have been developed to handle this problem of non-linear and linear programming [1], [2].

In addition, a more complicated model has been formulated by considering transportation costs [3], and it has dealt with known demand values [4]-[7]. Besides operational costs, some other models include other factors such as risk [8], [9]. Since some parameters are possibly uncertain, historical/experiment/trial data available can be treated as random variables with appropriate probability distribution functions. Therefore, the corresponding mathematical optimization problems can be solved through stochastic programming [10]-[12].

Subsequently, the probability distribution function cannot be formulated when the data needed is unobtainable. Then uncertain parameters are possibly treated as fuzzy variables, and membership functions can replace the probability distribution. The corresponding mathematical optimization problem is probably solved through fuzzy programming, which offers algorithms to handle optimization problems containing fuzzy variables [13]. This article has offered a newly developed mathematical optimization model in a fuzzy programming form to optimize raw parts procurement and inventory management under excess demand situations and fuzzy prices due to competition.

Furthermore, this model can be suitably used in postpandemic cases, and the numerical experiment results will be discussed to demonstrate the proposed model.

The remaining sections of this article are structured as follows. In Section II, the research gap studied in this research is described. In Section III, the methodology implemented in the study is specified together with the assumptions. Furthermore, the mathematical model proposed in the study is also provided in this section. In Section IV, results of the numerical experiments carried out in this research are reported. Finally, the last section concludes the study and provides some outlooks for future researches.

II. Literature Review

II.1. Covid-19 Pandemic and Challenges in Supply Chain Management

Marcus Vinicius Dantas de Assunção [14] has stated that running a business during the pandemic or postpandemic situations has several challenges. First, prices are uncertain, and historical data do not fit the behavior of the prices. For example, transportation cost, raw material price, and operational/production cost tend to have different behavior during or in post-pandemic situations. Therefore, a different approach is needed in determining optimal decisions. Second, the capacities of some parties are highly fluctuated and consequently uncertain. For example, suppliers' capabilities and production units can suddenly change due to infections.

Third, demand for some commodities is excess [15].

For example, the demand for many healthcare products is usually in excess during a pandemic [16]. Fourth, a "travel boom" phenomenon is predicted shortly after a pandemic [17]. This will affect some commodities in excess demand situations, such as tourism and hospitality-related products. Therefore, organizationallevel responses are needed in public and private firms [18]-[21]. Furthermore, the decision-maker should prepare a suitable tool to gain profit extensively by making optimal decisions in this situation. It has been reported that supply chain management can mitigate the effects of disruptions impacts during a pandemic [22], [23]. Many studies conducted in many fields are summarized in Table I, where each approach implemented a unique technique following the problem's specifications considered in the study. In addition, the specifications have showed the differences and the contributions to this study.

II.2. Research Gap

Many decision-making support tools have been developed in challenging pandemic situations (Table I).

Each decision-making support has been built based on a specific problem faced by the decision-maker. As a result, some studies have coincident specifications, as shown in Table I, while others have gaps. The type discussed has been the integrated procurement problem and the inventory management in an uncertain environment where many parameters' values are unknown. Moreover, the paper has also dealt with multiperiod (dynamic over time), multi-product, multisupplier situations.

TABLE I	
STUDIES REGARDING PROCUREMENT PROBLEM AND INVENTORY PROBLEM DURING/AFTER PANDEMIC	

Article	Procurement Problem	Inventory Problem	Other problems	Dynamic/Multi- period	Multi- product	Multi- supplier	Uncertain environment	Method to solve
[24]	yes	no	-	no	no	yes	yes	Simheuristic Approach Based on the Location-Routing Problem
[25]	Yes	yes	-	no	no	yes	yes	Derivative-based optimization
[26]	no	yes	-	yes	no	no	no	Tabu search
[27]	no	no	TSP	no	yes	no	yes	Ant Colony Optimization
[28]	yes	yes	disruption risk mitigation	yes	no	no	yes	Genetic algorithm and pattern search
[29]	no	yes	-	yes	no	no	no	Convex-function-based optimization
[30]	yes	yes	-	yes	yes	yes	no	Multi-Objective Optimization
This paper	yes	yes	Excess demand situations	yes	yes	yes	yes	Fuzzy programming

III. Materials and Methods

III.1. Assumptions

The following assumptions are considered:

- 1. Shortages are not allowed;
- 2. Extra or penalty costs for defective parts and late delivered parts are charged to the manager and not the suppliers. The penalty cost for faulty parts may be cheaper than the original price since they can be repaired or reused. Meanwhile, the penalty cost for late delivered parts occurs since the manager may still spend the cost for the production department.

III.2. Methodology

Fig. 1 shows the solution procedure used, and in the first three steps, the DM has a significant role in defining the membership function value for each fuzzy parameter.

This defining process uses intuition from the DM based on experience. Let \tilde{D}_{tp} denote the fuzzy variable of demand value of part p at review period t. The solution needs to satisfy the importance of demand, including the total purchased part that should be greater or equal to the demand value.

However, the purchase variables should be more significant when the demand is uncertain, which is approached by a fuzzy variable. This condition is unclear since the feasible set does not produce a crisp background.

Therefore, the optimal strategy cannot be determined as a crisp value. For a fuzzy variable to have a crisp value, the fuzzy demand has been approached by using the expected value. There are several formulas to calculate the expected value of a fuzzy number, and the proposed approach has used the expected value defined in [31], where the expectation of \tilde{D}_{tr} will be:

$$E\left[\tilde{D}_{tp}\right] = \int_0^\infty Cr\left\{\tilde{D}_{tp} \ge r\right\} dr - \int_{-\infty}^0 Cr\left\{\tilde{D}_{tp} \le r\right\} dr \quad (1)$$

for any period t and part p, provided that at least one result of these two integral terms is finite and $Cr[\cdot]$ denotes the credibility value.



Fig. 1. Solution procedure of the problem

The formula (1) calculates the expectation of any fuzzy number according to its membership function. For a particular case of discrete fuzzy number/variable ξ that has a membership function:

$$\mu_{\xi}(x) = \begin{cases} \mu_{1}, \text{ if } x = x_{1} \\ \mu_{2}, \text{ if } x = x_{2} \\ \vdots \\ \mu_{m}, \text{ if } x = x_{m} \end{cases}$$
(2)

where x_1, x_2, \dots, x_m are distinct and $x_m > x_{m-1} > \dots > x_2 > x_1$, the expectation of ξ will be:

$$E[\xi] = \sum_{i=1}^{m} w_i x_i \tag{3}$$

where for *i*=1,2,..., *m*:

$$w_i = \frac{1}{2} \left(\max_{1 \le j \le i} \mu_j - \max_{1 \le j < i} \mu_j + \max_{i \le j \le m} \mu_j - \max_{1 < j \le m} \mu_j \right) \quad (4)$$

III.3. Mathematical Model

The objective function is the expected total profit,

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including the total income minus the total operational costs maximized. Each mathematical optimization component is modeled as follows. First, the objective value represents the maximized expected profit, and it is the total income minus the total costs. Each component is formulated as:

1. The expected income value from selling products for all brands over the whole observation period:

$$F_1 = \sum_{\{t,b\} \in \mathcal{T} \times \mathcal{B}} \left(\widetilde{BP}_{tb} Y_{tb} \right)$$

2. The expected total cost for assembly/production processes and storing products has been observed:

$$F_{2} = \sum_{\{t,b\}\in\mathcal{T}\times\mathcal{B}} \left[\left(\widetilde{AC}_{tb}Y_{tb} \right) + \left(HB_{tb}i_{tb}^{Y} \right) \right]$$

3. Expected costs for buying the parts, including penalty costs for defective parts and delayed/late delivered parts. The first component represents the total cost for purchasing the parts, the second component represents the extra cost for faulty parts charged to the manager and not to the suppliers, and the third component represents the extra:

$$\begin{split} F_{3} &= \sum_{\{t,s,p\} \in \mathcal{T} \times \mathcal{S} \times \mathcal{P}} \left[\left(\widetilde{PP}_{tsp} X_{tsp} \right) + \left(PD_{tp} \widetilde{DR}_{tsp}^{X} X_{tsp} \right) + \right. \\ &\left. + \left(PL_{tp} \cdot \widetilde{LR}_{tsp} \cdot X_{tsp} \right) \right] \end{split}$$

4. The expected total order cost plus transportation cost from all selected suppliers over observation periods is:

$$F_4 = \sum_{\{t,s\}\in\mathcal{T}\times\mathcal{S}} \left[\left(\tilde{O}_{ts} Z_{ts} \right) + \left(\widetilde{TC}_{ts} S_{ts} \right) \right]$$

5. The expected total contract cost for all the selected suppliers is:

$$F_5 = \sum_{s \in \mathcal{S}} \left(\widetilde{NC}_s W_s \right)$$

6. The total inventory cost for all the parts stored in the warehouse over the observation periods is:

$$F_6 = \sum_{\{t,p\} \in \mathcal{T} \times \mathcal{P}} \left(HP_{tp} i_{tp}^X \right)$$

Second, constraint functions based on situations that should be hold are formulated as:

1. Constraints that manage the flow of the parts. They have stated that at a particular observation period t, for every part p, the parts from the previous observation period, those purchased from all suppliers, parts that just arrived due to late delivery,

minus parts purchased at the current observation period but will be delivered at the next, minus defected parts and those saved in the inventory to be used in the future observation time are the total numbers of components needed for making product brands at the current observation period:

$$\begin{split} i_{(t-1)p}^{X} + \sum_{s=1}^{S} \left[X_{tsp} + \left(LR_{(t-1)sp} X_{(t-1)sp} \right) + \\ - \left(\widetilde{LR}_{tsp} X_{tsp} \right) - \widetilde{DR}_{tsp} X_{tsp} \right] - i_{tp}^{X} \ge \\ \ge \sum_{\{p,b\} \in \mathcal{P} \times \mathcal{B}} \left(RP_{pb} Y_{tb} \right), \quad \forall \{t, p\} \in \mathcal{T} \times \mathcal{P} \end{split}$$

2. Product brand demands satisfying. The constraints say that the number of product brand b produced every observation period t minus defected ones minus some of them planned to be stored in the inventory is expected to satisfy the demand value at the corresponding observation period:

$$Y_{tb} - \left(\widetilde{DR}_{tb}^{Y}Y_{tb}\right) - i_{tb}^{Y} \ge \widetilde{D}_{tb}, \quad \forall \left\{t, b\right\} \in \mathcal{T} \times B$$

3. The constraints function determines the number of trucks used to transport parts from a particular supplier. The constraints say that the total volume of parts purchased at a specific supplier divided by the truck's maximum volume capacity, rounded to above integer, should be upper bounded by the number of the trucks. Therefore, the number of trucks will be minimized according to the transport cost function in F_4 :

$$\left\lceil \frac{\sum_{p=1}^{P} X_{tsp}}{C} \right\rceil \leq S_{ts}, \forall s \in S, \forall t \in T;$$

4. At every observation time, the number of each part purchased by a particular supplier cannot exceed the corresponding supplier's maximum capacity to supply the parts:

$$X_{tsp} \leq SC_{tsp}, \forall p \in P, \forall s \in S, \forall t \in T$$

5. Indicator functions represent whether a supplier is selected or not at a particular observation period *t*. For example, it will be 1 when the corresponding supplier is selected at the corresponding observation period *t* to supply some parts; otherwise, it will be 0, i.e., no order cost occurs:

$$Z_{ts} = \begin{cases} 1, & \text{if } \sum_{p=1}^{P} X_{tsp} > 0\\ 0, & \text{otherwise} \end{cases} \quad \forall \{t, s\} \in \mathcal{T} \times \mathcal{S}$$

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6. Indicator functions represent whether a supplier is selected at least once or not over observation periods 1 to *T*. It will be 1 when the corresponding supplier is chosen at least once to supply some parts; otherwise, it will be 0, i.e., no new supplier contract cost occurs:

$$W_{s} = \begin{cases} 1, & \text{if } \sum_{t \in \mathcal{T}} Z_{ts} > 0\\ 0, & \text{otherwise} \end{cases} \quad \forall s \in \mathcal{S};$$

7. The number of parts *b* planned to be stored in the inventory at every observation period cannot exceed the maximum capacity of its warehouse/storing unit:

$$i_{tp}^{X} \le MP_{tp}, \forall p \in P, \forall t \in T$$

8. The number of products brand *b* planned to be stored in the inventory at every observation period cannot exceed the maximum capacity of its warehouse/storing unit:

$$i_{tb}^{Y} \leq MB_{tb} \quad \forall \{t, b\} \in \mathcal{T} \times B$$

9. The decision variables should be nonnegative and integer:

$$\left\{X_{tsp}, Y_{tb}, i_{tp}^{X}, i_{tb}^{Y}\right\} \ge 0$$

and integer $\forall \{t, s, p, b\} \in \mathcal{T} \times \mathcal{S} \times \mathcal{P} \times \mathcal{B}$

This is modeled completely as:

$$\max_{\substack{\{X_{xsp}, Y_{tb}, i_{tp}^{X}, j_{tb}^{Y}\}, \\ \{t, s, p, b\} \in \mathcal{T} \times \mathcal{S} \times \mathcal{P} \times \mathcal{B}}} (F_{1} - F_{2} - F_{3} - F_{4} - F_{5} - F_{6})$$
(5)

subject to:

$$i_{(t-1)p}^{X} + \sum_{s=1}^{S} \left[X_{tsp} + \left(LR_{(t-1)sp} X_{(t-1)sp} \right) + \left(\widetilde{LR}_{tsp} X_{tsp} \right) - \widetilde{DR}_{tsp} X_{tsp} \right] - i_{tp}^{X} \ge$$

$$(6)$$

$$\geq \sum_{\{p,b\}\in\mathcal{P}\times\mathcal{B}} \left(RP_{pb}Y_{tb} \right), \quad \forall \{t,p\}\in\mathcal{T}\times\mathcal{P}$$

$$Y_{lb} - \left(\widetilde{DR}_{lb}^{Y}Y_{lb}\right) - i_{lb}^{Y} \ge \widetilde{D}_{lb}, \quad \forall \{t, b\} \in \mathcal{T} \times B$$
(7)

$$\left[\frac{\sum_{p=1}^{P} X_{tsp}}{C}\right] \le S_{ts}, \forall s \in S, \forall t \in T$$
(8)

$$X_{tsp} \le SC_{tsp}, \forall p \in P, \forall s \in S, \forall t \in T$$
(9)

$$Z_{ts} = \begin{cases} 1, & \text{if } \sum_{p=1}^{P} X_{tsp} > 0\\ 0, & \text{otherwise} \end{cases} \quad \forall \{t, s\} \in \mathcal{T} \times \mathcal{S} \qquad (10)$$

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$$W_{s} = \begin{cases} 1, & \text{if } \sum_{t \in \mathcal{T}} Z_{ts} > 0\\ 0, & \text{otherwise} \end{cases} \quad \forall s \in \mathcal{S}$$
(11)

$$i_{tp}^{X} \le MP_{tp}, \forall p \in P, \forall t \in T$$
(12)

$$i_{tb}^{Y} \le MB_{tb} \quad \forall \{t, b\} \in \mathcal{T} \times B$$
 (13)

$$\left\{X_{tsp}, Y_{tb}, i_{tp}^{X}, i_{tb}^{Y}\right\} \ge 0$$

and integer $\forall \{t, s, p, b\} \in \mathcal{T} \times \mathcal{S} \times \mathcal{P} \times \mathcal{B}$ (14)

IV. Computational Simulation Results

IV.1. Parameter Setup

The optimization problem (5) contains three part types ordered to four suppliers, and these parts can be used to make four product brands. First, let P1, P2, and P3 be the name of the parts. Subsequently, the names of the suppliers are S1, S2, S3, and S4, while the product brands are B1, B2, B3, and B4. Finally, the following discrete membership functions for the fuzzy parameters contained in the problem is considered:

$$\mu_{\xi} = \begin{cases} \mu_{\xi^{(1)}} & \text{if } \xi = \xi^{(1)} \\ \mu_{\xi^{(2)}} & \text{if } \xi = \xi^{(2)} \\ \vdots \\ \mu_{\xi^{(10)}} & \text{if } \xi = \xi^{(10)} \end{cases}$$

The values for components in the membership function above are shown in Tables A1 to A10, while the values for the crisp parameters are given in Tables A11 to A16, where the truck's capacity is 200 units.

IV.2. Results and Discussion

All the computations have been performed in a standard personal computer by using LINGO optimization software. Therefore, the computational time has not been an issue since it has been done in minutes.

Results are shown in Figures 2 and 3, and the minimal value of the objective function is 4,048,618.



Fig. 2. Optimal part volume for periods 1 to 2



Fig. 3. Optimal part volume for Example 2

Figure 2 shows the optimal volume of parts to be ordered by suppliers. Due to high demand, all the suppliers have been selected to supply parts by using their maximum capacity. Figure 3 shows the volume of each product brand planned to be produced. All the product brands have satisfied the demands since the decisions regarding the inventory have not been storing any parts P1 and P3 at observation periods 1 and 2. For example, 100 units part type P2 needs to be stored in the warehouse at the observation period 1 to be used at period 2. Therefore, no inventory is required at any observation time for the products. Drawing upon the mathematical model presented in the preceding section and the results of the associated numerical experiments, the following discussion offers managerial insights that can serve as valuable advice for industry managers and decision-makers. Firstly, managers have the flexibility to fine-tune the membership functions of fuzzy parameters and recompute decisions, if they have the time for computational adjustments. This becomes particularly relevant when dealing with sizable problems that naturally require extended computational durations.

Additionally, managers can explore alternative membership function shapes, such as triangular or trapezoidal, tailored to specific fuzzy parameters.

Secondly, in order to address computational challenges, managers can opt for computers equipped with more robust specifications, including high-performance machines. This choice can significantly reduce computational time, proving especially advantageous for tackling large-scale problems. Lastly, it is important to acknowledge that the problem is resolved amid uncertainties surrounding certain parameters.

Consequently, the actual profit may deviate from the optimization result. The true profit will only be known once all uncertain parameters are revealed. Nonetheless, optimizing decisions under such conditions represents the best course of action available to managers when confronted with problems involving uncertainties.

V. Conclusion

A fuzzy-based optimization model has been proposed to solve part procurement and inventory management problems where several parameters have been treated as fuzzy parameters. The main advantage of the proposed model is that the problems are solved in an integrated manner, meaning that the parties involved in the supply chain are treated via interconnected scheme such that the flow of the raw parts from the suppliers to the inventory systems is managed in one model. Then the optimal decision is the best scenario for the whole supply chain.

Furthermore, the proposed fuzzy-uncertainty approach can handle difficulties with excess demand and uncertain parameters, which are suitable for extra-ordinary situations such as post-pandemic time. Numerical experiment results have showed that the proposed decision-making support successfully solved the given problem. Therefore, it can be used in industrial/retail industries. For future studies, the model can be developed under data availability assumptions towards the end of the pandemic. In this situation, parameters already have historical data, and therefore, probability theory can be used to treat the uncertain parameters.

Another possible future research direction could be integrating other parties that are not involved yet in this study, such as third party carriers and distributors. In particular, in terms of optimization methods, metaheuristic algorithms could be developed to solve large-scale problems, for example, problems with large number of suppliers and raw part types.

Appendix

TABLE A1 VALUES FOR μ_{-}

		new			
i	$\mu_{\widetilde{AC}^{(l)}_{\scriptscriptstyle B}}$	\widetilde{AC}_{ts}	$\mathcal{W}^{(i)}_{\widetilde{AC}_{ss}}$		
1	20	0.10	0.050		
2	25	0.20	0.050		
3	30	0.35	0.075		
4	35	0.50	0.075		
5	40	0.75	0.125		
6	45	1.00	0.250		
7	50	0.75	0.150		
8	55	0.45	0.100		
9	60	0.25	0.050		
10	65	0.15	0.075		
TABLE A2 VALUES FOR $\mu_{\widetilde{BP}_{\phi}}$					
		\widetilde{RP}_{th}	$w_{\widetilde{\sim}}^{(i)}$		
1	$\mu_{\widetilde{BP}_{tb}}^{(i)}$	D1 10	BP_{ib}		
1	$\frac{\mu_{\widetilde{BP}_{tb}^{(l)}}}{1000}$	0.20	0.100		
1 1 2	$\frac{\mu_{\widetilde{BP}_{tb}^{(l)}}}{1000}$	0.20 0.35	0.100 0.075		
	$\frac{\mu_{\widehat{BP}_{th}^{(i)}}}{1000} \\ 1100 \\ 1200$	0.20 0.35 0.50	$ \begin{array}{r} 0.100 \\ 0.075 \\ 0.075 \end{array} $		
	$\frac{\mu_{\widehat{BP}_{th}^{(i)}}}{1000} \\ 1100 \\ 1200 \\ 1300$	0.20 0.35 0.50 0.65	0.100 0.075 0.075 0.075		
1 2 3 4 5	$ \frac{\mu_{BP_{b}^{(i)}}}{1000} 1100 1200 1300 1400 $	0.20 0.35 0.50 0.65 0.70	$\begin{array}{c} 0.100\\ 0.075\\ 0.075\\ 0.075\\ 0.025\\ \end{array}$		
$ \begin{array}{c} 1\\ 2\\ 3\\ 4\\ 5\\ 6 \end{array} $	$\frac{\mu_{\bar{B}\bar{P}_{0}^{(i)}}}{1000}$ 1000 1100 1200 1300 1400 1500	0.20 0.35 0.50 0.65 0.70 0.85	$\begin{array}{c} 0.100\\ 0.075\\ 0.075\\ 0.075\\ 0.025\\ 0.025\\ 0.075\\ \end{array}$		
1 2 3 4 5 6 7	$\frac{\mu_{BP_{0}^{(i)}}}{1000}$ 1000 1100 1200 1300 1400 1500 1600	0.20 0.35 0.50 0.65 0.70 0.85 0.95	$\begin{array}{c} 0.100\\ 0.075\\ 0.075\\ 0.075\\ 0.025\\ 0.075\\ 0.050\\ \end{array}$		
1 2 3 4 5 6 7 8	$\begin{array}{c} \mu_{\widehat{BP}_{0}^{(i)}} \\ \hline 1000 \\ 1100 \\ 1200 \\ 1300 \\ 1400 \\ 1500 \\ 1600 \\ 1700 \end{array}$	0.20 0.35 0.50 0.65 0.70 0.85 0.95 1.00	$\begin{array}{c} 0.100\\ 0.075\\ 0.075\\ 0.075\\ 0.025\\ 0.075\\ 0.050\\ 0.150\\ \end{array}$		
1 2 3 4 5 6 7 8 9	$\begin{array}{c} \mu_{\widehat{BP}_{0}^{(i)}} \\ \hline 1000 \\ 1100 \\ 1200 \\ 1300 \\ 1400 \\ 1500 \\ 1600 \\ 1700 \\ 1800 \end{array}$	0.20 0.35 0.50 0.65 0.70 0.85 0.95 1.00 0.75	$\begin{array}{c} 0.100\\ 0.075\\ 0.075\\ 0.075\\ 0.025\\ 0.075\\ 0.050\\ 0.150\\ 0.125\\ \end{array}$		
1 2 3 4 5 6 7 8 9 10	$\begin{array}{c} \mu_{\widehat{B}\widehat{P}_{B}^{(r)}} \\ \hline 1000 \\ 1100 \\ 1200 \\ 1300 \\ 1400 \\ 1500 \\ 1600 \\ 1700 \\ 1800 \\ 1900 \\ \end{array}$	0.20 0.35 0.50 0.65 0.70 0.85 0.95 1.00 0.75 0.50	$\begin{array}{c} 0.100\\ 0.075\\ 0.075\\ 0.075\\ 0.025\\ 0.075\\ 0.050\\ 0.150\\ 0.125\\ 0.250\\ \end{array}$		
1 2 3 4 5 6 7 8 9 10	μ _{BP₀} ⁽ⁱ⁾ 1000 1100 1200 1300 1400 1500 1600 1700 1800 1900 TAI VALUES	$\begin{array}{c} 0.20\\ 0.35\\ 0.50\\ 0.65\\ 0.70\\ 0.85\\ 0.95\\ 1.00\\ 0.75\\ 0.50\\ \end{array}$ BLE A3 FOR $\mu_{\widetilde{P}\widetilde{P}_{rp}}$	$\begin{array}{c} & & & & \\ & & & & \\ 0.100 \\ & & & & \\ 0.075 \\ & & & & \\ 0.075 \\ & & & & \\ 0.075 \\ & & & & \\ 0.075 \\ & & & & \\ 0.075 \\ & & & & \\ 0.075 \\$		

i	$\mu_{\widetilde{PP}^{(l)}_{tsp}}$	\widetilde{PP}_{tsp}	$\mathcal{W}^{(i)}_{\widetilde{PP}_{tsp}}$
1	15	0.25	0.125
2	16	0.50	0.125
3	17	0.75	0.125
4	18	0.85	0.050
5	19	0.95	0.050
6	20	1.00	0.100
7	21	0.85	0.050
8	22	0.75	0.125
9	23	0.50	0.125
10	24	0.25	0.125

	TAH Values	BLE A4 For $\mu_{\widetilde{TC}_{ss}}$		TABLE A5 Values For $\mu_{\tilde{O}_i}$			
i	$\mu_{\widetilde{\mathit{TC}}^{(i)}_{\mathit{ss}}}$	\widetilde{TC}_{ts}	$\mathcal{W}_{\widetilde{TC}_{B}}^{(i)}$	i	$\mu_{\widetilde{O}^{(l)}_s}$	$ ilde{O}_{s}$	$w^{(i)}_{ ilde{O}_s}$
1	200	0.15	0.075	1	41	0.35	0.175
2	210	0.45	0.150	2	42	0.40	0.025
3	220	0.65	0.100	3	43	0.50	0.050
4	230	0.75	0.050	4	44	0.77	0.135
5	240	0.95	0.100	5	45	0.85	0.040
6	250	1.00	0.050	6	46	1.00	0.125
7	260	0.95	0.115	7	47	0.90	0.050
8	270	0.72	0.085	8	48	0.80	0.150
9	280	0.55	0.165	9	49	0.50	0.200
10	290	0.22	0.110	10	50	0.10	0.050
	TAI Values	BLE A6 FOR $\mu_{\widetilde{LR}_{NR}}$			TAI Values	BLE A7 S For $\mu_{\widetilde{AC}_{+}}$	
i	$\mu_{\sim \omega}$	ĨR.	$w_{\widetilde{\sim}}^{(i)}$	i	$\mu_{\sim \infty}$	\widetilde{DD}^X	$w^{(i)}$
1		LIK isp		i	$DR_{tsp}^{(1)}$	DK_{tsp}	\widetilde{DR}_{tsp}^{A}
1	0.01	0.15	0.075	1	0.01	0.25	0.125
2	0.02	0.25	0.050	2	0.02	0.40	0.075
3	0.03	0.45	0.100	3	0.03	0.50	0.050
4	0.04	0.55	0.050	4	0.04	0.75	0.125
5	0.05	0.75	0.100	5	0.05	0.95	0.100
6	0.06	1.00	0.150	6	0.06	1.00	0.100
7	0.07	0.95	0.125	7	0.07	0.85	0.150
8	0.08	0.70	0.225	8	0.08	0.55	0.050
9	0.09	0.25	0.075	9	0.09	0.45	0.100
10	0.1	0.10	0.050	10	0.1	0.25	0.125
	TAI	BLE A8			TAI	BLE A9	
	VALUES	FOR $\mu_{\widetilde{DR}^{Y}}$			VALUES	SFOR $\mu_{\widetilde{NC}_s}$	
i	$\mu_{\widetilde{DRY}^{(i)}_{tb}}$	\widetilde{DR}_{tb}^{Y}	$W^{(i)}_{\widetilde{DR}^Y_{\phi}}$	i	$\mu_{_{\widetilde{NC}^{(l)}_{s}}}$	\widetilde{NC}_s	$\mathcal{W}_{\overline{NC}_s}^{(i)}$
1	0.01	0.15	0.075	1	100	0.20	0.100
2	0.02	0.45	0.150	2	110	0.45	0.125
3	0.02	0.80	0.175	3	120	0.55	0.050
4	0.04	1.00	0.125	4	130	0.75	0.100
5	0.05	0.95	0.050	5	140	1.00	0.150
6	0.06	0.85	0.050	6	150	0.95	0.050
7	0.07	0.75	0.100	7	160	0.85	0.140
8	0.08	0.55	0.175	8	170	0.57	0.160
9	0.09	0.20	0.025	9	180	0.25	0.050
10	0.1	0.15	0.075	10	190	0.15	0.075

TABLE A10 VALUES FOR $\mu_{\tilde{D}}$.

		· D _{tb}	
i	$\mu_{\widetilde{D}_{tb}^{(i)}}$	$\mu_{\widetilde{D}_{tb}}$	$w^{(i)}_{\mu_{\mathcal{D}_{bb}}}$
1	80	0.20	0.100
2	85	0.55	0.175
3	90	0.75	0.100
4	95	1.00	0.150
5	100	0.95	0.075
6	105	0.80	0.050
7	110	0.70	0.150
8	115	0.40	0.090
9	120	0.22	0.060
10	125	0.10	0.050

TABLE A11 VALUES FOR *RP*_{pb} Product Brand B1 B2 B3

Dout trues	Product Brand			
Part type	B1	B2	B3	B4
P1	2	1	3	1
P2	1	2	1	2
P3	2	1	2	2

	SUPPL	IER CAPACITY VALUES	SSC_{tsp}		
Supplier		D1	Part tvn	5	D2
S1 S2 S3 S4		P1 P2 800 450 950 500 1000 450 750 500		50 00 50 00	650 850 1000 1200
		TABLE A13 VALUES FOR RP_{pb}			
Paran	neter	Р	21	P2	P3
Holding o Warehouse c	cost <i>HP</i> _{tp} anacity <i>MP</i> _{tp}	8	1 80	1 100	$\frac{2}{100}$
	Brands' Par	TABLE A14 AMETER VALUES FOR	All Periods		
Parameter		B1	B2	B3	B4
Holding cost <i>H</i>	IP_{tp}	3	2	2	2
warehouse capaci	ty MP_{tp}	2000	2000	2000	2000
		TABLE A15VALUES FOR PD^{X}_{tp}			
Supplier	<u>P1</u>	<u>P2</u>		<u>P3</u>	
S1 S2	$\frac{1}{2}$	2		l 1	
S2 S3	1	2		2	
S4	3	2		1	
		TABLE A16 Values For <i>Pl</i> _{1p}			
Supplier	P1		P2		P3
S1	1		1		2
52 53	1		2		1
S4	2		1		1

TABLE A12 SUPPLIER CAPACITY VALUES SC₁₅₀

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